Society of Actuaries in Ireland

Probability- Hints and Tips

1. Probability of Events:

1.1 Probability of one event:

- Probability of event E, P(E) = $\frac{Number\ of\ outcomes\ satisfying\ E}{Total\ number\ of\ outcomes}$
- 0 ≤ P(E) ≤ 1, meaning that probabilities cannot be negative or greater than 1.
- P(E) = 0 means that event E can't happen.
- P(E) = 1 means event E is certain to happen.
- P(E') is the probability that E does not happen.

1.2 Probability of multiple events:

- P (A ∩ B) is the probability that events A and B happen at the same time.
- P (A' ∩ B) is the probability that B happens but A does not happen at the same time
- P (A or B) = P(A) + P(B) – P(A ∩ B).

1.3 Conditional Probability:

- P (A|B) is the probability that A happens given that B has happened.

1.4 Independent Events:

- Two events, A and B, are independent if the occurrence of one event does not affect the occurrence of the other e.g. flipping a coin and getting tails (A) and rolling a six-sided die and getting a 2 (B).
- For independent events, P (A ∩ B) = P(A) * P(B).
- Similarly for independent events:
  - P (A|B) = P (A ∩ B) / P(B) = P(A) * P(B) / P(B) = P(A)
  - P(A ∩ B) = P(A) * P(B|A) = P(A) * P(B)
  - P(A ∩ B) = P(B) * P(A|B) = P(B) * P(A)

1.5 Expected Value:

- The expected value of a random variable is the sum of all the values that the random variable can take multiplied by the probability of the random variable being equal to that value.
- It is another word for the average value of the random variable.
- Formula is given by: E(X) = Σx*P(X=x)

2. Combinations and Permutations

- **Permutations:** The number of ways to arrange r objects from a set of n distinct objects. P(n,r)=$\frac{n!}{(n-r)!}$. Whenever the question mentions arrangements, think of permutations.
- **Combinations:** The number of ways to choose r objects from a set of n distinct objects, regardless of the order. C(n,r) = $\frac{n!}{r!(n-r)!}$. Whenever the question mentions choosing or choices, think of combinations.
- You need to be able to use your calculator to calculate different forms of permutations and combinations.
- To do this, first select the number n on your calculator, followed by the nCr button followed by the number r.

Probability- Hints and Tips

- E.g. There are 11 students, how many choices of 5 a side teams can be made from those 11 students.
- In this question, the order does not matter, and we are looking for number of choices, so we are dealing with combinations.
- To calculate the number of combinations, we use our calculator and input 11, nCr, 5 and get an answer of 462.

3. Bernoulli Trials

3.1 Bernoulli Trials

- A Bernoulli trial is one that only has two outcomes, success or failure.
- The trials are independent.
- Probability of success is p.
- Probability of failure is (1-p).

3.2 Probability of k success in n Bernoulli Trials

- The formula for calculating the probability of r success in n events is given by the formula on page 33 of the log tables:

Dáiltí dóchúlachta		**Probability distributions**
an dáileadh déthéarmach	$P(X=r)=\binom{n}{r}p^r q^{n-r}$ $r=0\ldots n$	binomial distribution

- E.g. A fair die is thrown 5 times. Calculate the probability of getting 3 fours to 3 decimal places.
  - Probability of getting a four = $\frac{1}{6}$
  - Probability of not getting a four = $\frac{5}{6}$
  - Probability of 3 fours = ${}_5C_3 * \left(\frac{1}{6}\right)^3 * \left(\frac{5}{6}\right)^2 = 0.032$

3.3 Probability that the kth success occurs on the nth Bernoulli Trial

- In order for the kth success to happen on the nth trial, that means there must have been (k-1) success in (n-1) trials followed by a success on the nth trial.
- The formula is given by ${}_{(n-1)}C_{(k-1)} * p^k * (1-p)^{(n-k)}$.
- E.g. A card is randomly selected from a deck of cards and then placed back. Calculate the probability that the 3rd diamond is selected on the 10th card.
  - First calculate the probability of selecting 2 diamonds from the first 9 cards
  - ${}_9C_2 * \left(\frac{1}{4}\right)^2 * \left(\frac{3}{4}\right)^7 = 0.3$
  - Then multiply this by the probability of getting another diamond.
  - = 0.3 * 0.25 = 0.075.

