Society of Actuaries in Ireland

Calculus 2 Tutorial Questions

Exercises:

1) Integrate the following:

i. $\int 4x\,dx$ ii. $\int 5y^3\,dy$ iii. $\int 7x^5 + 6x + 2\,dx$ iv. $\int \left(\frac{1}{x}\right)\,dx$

2) Calculate the area under the curve $f(x) = 6x^2$ over the interval [2,4].

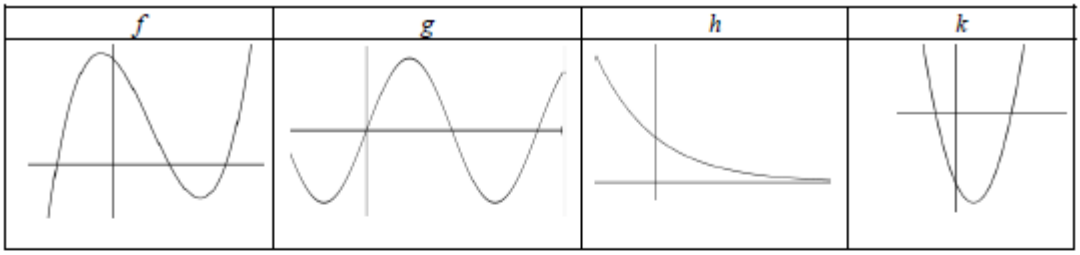
3) Calculate the area under the curve $f(x) = 4x^3 + 2$ over the interval [1,5].

4) Calculate the average value of the function $f(x) = 5 - 2x$ over the interval [0,3].

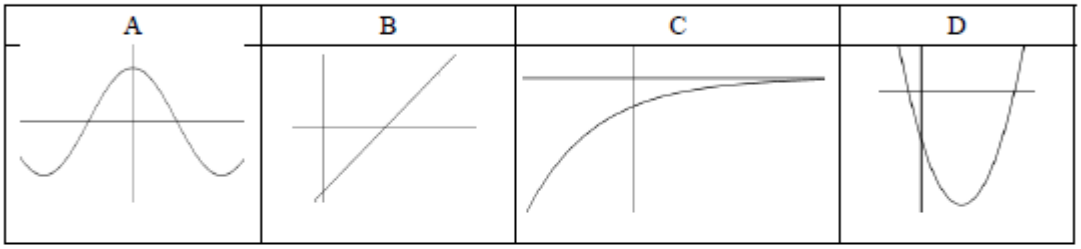
Past Exam Questions

Question 1 [2013 Sample Paper 1, Q5] (25 marks)

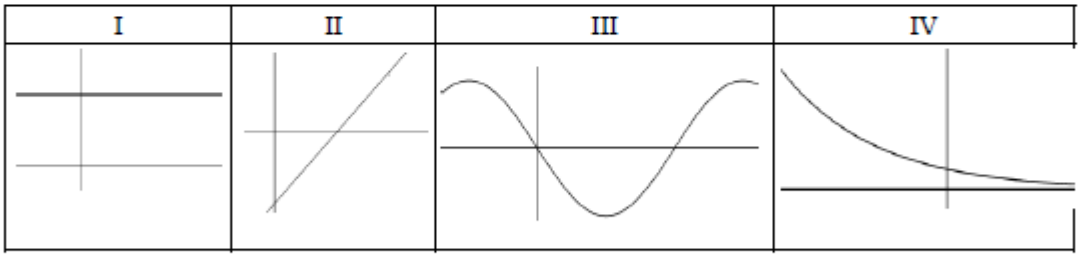
Each diagram below shows part of the graph of a function. Each of these functions is either quadratic or cubic or trigonometric or exponential (not necessarily in that order).



Each diagram below shows part of the graph of the first derivative of one of the above functions (not necessarily in the same order).



Each diagram below shows part of the graph of the second derivative of one of the original functions (not necessarily in the same order).



(a) Complete the table below by matching the function to its first derivative and its second derivative.

Type of function	Function	First derivative	Second derivative
Quadratic			
Cubic			
Trigonometric			
Exponential			

(b) For **one** row in the table, explain your choice of first derivative and second derivative.

Calculus 2 Tutorial Questions

Question 2 [2014 Paper 1, Q5] (25 marks)

(a) Find $\int 5\cos 3x\,dx$.

(b) The slope of the tangent to a curve $y = f(x)$ at each point (x, y) is $2x-2$. The curve cuts the x-axis at $(-2, 0)$.

(i) Find the equation of $f(x)$.

(ii) Find the average value of f over the interval $0 \le x \le 3, x \in \mathbb{R}$.

Question 3 [2015 Paper 1, Q3] (25 marks)

Let $f(x) = -x^2 + 12x - 27, x \in \mathbb{R}$.

(a) **(i)** Complete Table 1 below.

Table 1							
x	3	4	5	6	7	8	9
$f(x)$	0	5			8		

(ii) Use Table 1 and the trapezoidal rule to find the approximate area of the region bounded by the graph of f and the x-axis.

(b) **(i)** Find $\int_3^9 f(x)dx$.

(ii) Use your answers above to find the percentage error in your approximation of the area, correct to one decimal place.

Question 4 [2016 Paper 1, Q7] (40 marks)

(a) **(i)** Air is pumped into a spherical exercise ball at the rate of 250 cm^3 per second. Find the rate at which the radius is increasing when the radius of the ball is 20 cm. Give your answer in terms of π.

(ii) Find the rate at which the surface area of the ball is increasing when the radius of the ball is 20 cm.

(b) The inflated ball is kicked into the air from a point O on the ground. Taking O as the origin, $(x,\ f(x))$ approximately describes the path followed by the ball in the air, where

$$f(x) = -x^2 + 10x$$

and both x and $f(x)$ are measured in metres.

(i) Find the values of x when the ball is on the ground.

(ii) Find the average height of the ball above the ground, during the interval from when it is kicked until it hits the ground again.

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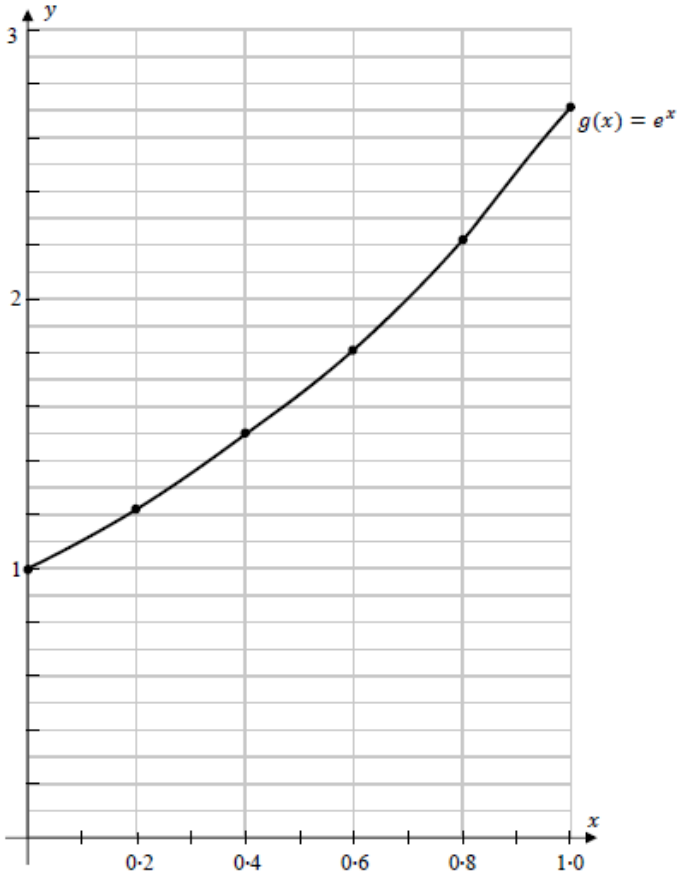
Question 5 [2017 Paper 1, Q6] (25 marks)

(a) The graph of the function g(x) = e^x, $0 \leq x \leq 1$is shown on the diagram below. On the same diagram, draw the graph of h(x) = e^{-x} in the domain $0 \leq x \leq 1$.

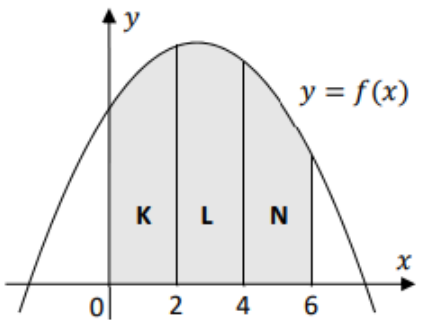


(b) Find the area enclosed by g(x) = e^x, h(x) = e^{-x} and the line x = 0.75. Give your answer correct to 4 decimal places.

Question 6 [2022 Paper 1, Q2] (15 marks)

(a) g(x) = $2x^2 + 5x + 6$. Find $\int g(x)\,dx$

(b) The diagram shows the graph of f(x) = $ax^2 + bx + c$. Three regions on the diagram are marked **K**, **L**, and **N**. Each of these regions is bounded by the x-axis, the graph of f(x) and 2 vertical lines.



The area of region **K** is 538 square units. Use integration of f(x) to show that 4a + 3b + 3c = 807.

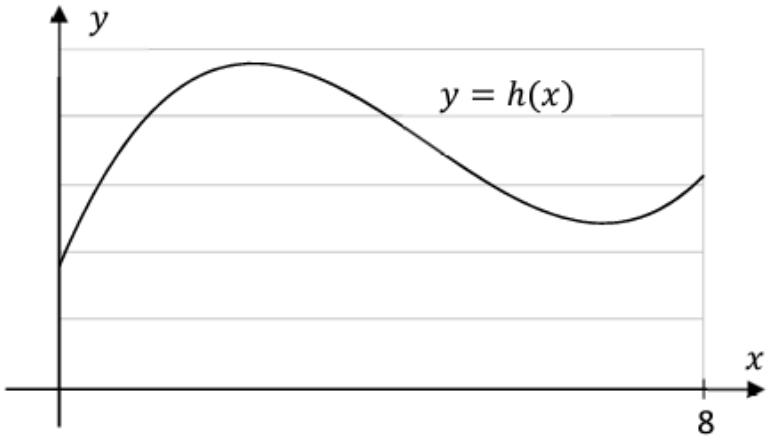
Calculus 2 Tutorial Questions

Question 7 [2022 Paper 1, Q7] (45 marks)

Hannah is doing a training session. During this sessions, h(x) is measured in beaths per minutes (BPM), where x is the time in minutes from the stat of the session. For the first 8 minutes, her heart rate changes. In this time h(x) is given by $h(x) = 2x^3 – 28.5x^2 + 105x + 70$

(a) Work out Hannah's heart rate 4 minutes after the start of the session.
(b) Find h'(x)
(c) Find h'(2) and explain what this value means in the context of Hannah's heart-rate.

The graph below shows y = h(x), where $0 \leq x \leq 8$.



(d) Find the least value and the greatest value of h(x) for $0 \leq x \leq 8$. Use calculus in your solution. You may also use information from the graph above, which is to scale.
(e) How long after the start of the session is Hannah's heart rate decreasing most quickly, within the first 8 minutes. Give your answer in minutes and seconds.

Bruno, Karen, and Martha start a training session at the same time as Hannah. All of their heartrates are measured in BP.

(f) (i) for the first 8 minutes of the session, Bruno's heart rate, b(x) is always 15BPM more than Hannah's heart rate. Use this information to write b'(x) **in terms of h'(x)**, where $0 \leq x \leq 8$.
(ii) For the first 8 minutes of the session, Karen's heart rate, k(x) is always 10% less than Hannah's heart rate. Use this information to write k'(x) **in terms of h'(x)**, where $0 \leq x \leq 8$.

