Society of Actuaries in Ireland

Calculus 2- Hints and Tips

2. Integration

2.1 Definition

- Integration is the reverse process of differentiation.
- The integral of a function between two points gives the **area** under the curve and above the x-axis, between those two points.

2.2 Notation

- If y = f(x), the integral is denoted as $\int f(x)\, dx$

2.3 Rules of Integration

2.3.1 Power Rule

- If f(x) = x^n then $\int f(x)\, dx = \frac{x^{(n+1)}}{n+1}$ + C where C is constant of integration. (Increase the power by 1 and divide by the power plus 1).
- The only exception to the above rule is n=-1 i.e. x^{-1}. The integral of f(x) = x^{-1} is ln(x) (Opposite of differentiation).
- E.g. If f(x) = $3x^3$ then $\int f(x) = \frac{3x^4}{4}$ + C

2.3.2 Sum/Difference Rule

- $\int \big(f(x) + g(x)\big)\, dx = \int f(x)\, dx + \int g(x)\, dx$
- Similarly $\int \big(f(x) - g(x)\big)\, dx = \int f(x)\, dx - \int g(x)\, dx$
- You can either combine similar terms together first and then integrate or integrate each function separately and then combine.
- E.g. f(x) = 5 – $3x^2$ and g(x) = $6x^2$ – 2x and we want to find $\int \big(f(x) - g(x)\big)\, dx$
  - Method 1 – Combine similar terms and then integrate. f(x) – g(x) = (5 – $3x^2$) – ($6x^2$ – 2x) = 5 – $9x^2$ + 2x
  - $\int 5 - 9x^2 + 2x\, dx = 5x - 3x^3 + x^2$
  - Method 2 – Integrate and then combine similar terms.
  - $\int f(x)\, dx = \int 5 - 3x^2\, dx = 5x - x^3$
  - $\int g(x)\, dx = \int 6x^2 - 2x\, dx = 2x^3 - x^2$
  - $\int f(x)\, dx - \int g(x)\, dx = 5x - x^3 - (2x^3 - x^2) = 5x - 3x^3 + x^2$

2.4 Common Integrals of Functions (*Page 26 of log tables*)

Function	Integral
$\frac{1}{x}$	Ln(x)
e^x	e^x
e^{ax}	$\frac{1}{a}e^{ax}$
Sin(x)	-Cos(x)
Cos(x)	Sin(x)
Tan(x)	Ln |sec(x)|

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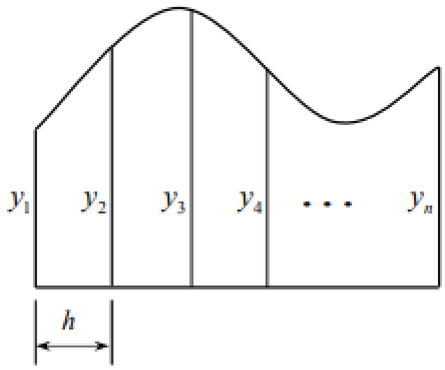
2.5 Area under a Curve

2.5.1 Trapezoidal Rule

- The Trapezoidal Rule is used to estimate the area under a curve.
- The formula is on page 12 of log tables: A $\approx \frac{h}{2}[y_1 + y_n + 2(y_2 + y_3 + \cdots y_{n-1})]$



- h is the difference between consecutive x values.

2.5.2 Integrate Function over Interval

- The Trapezoidal rule is just used to estimate the area under a curve.
- To calculate the actual area under a curve between an interval [a,b] we:
  - i. Integrate the function and ignore the constant C.
  - ii. Calculate the integral at the upper interval (b) and subtract the integral at the lower interval (a).
- E.g. Calculate the area under the curve f(x) = $3x^2$ between (2,5).
  - Integrate the function: $\int_2^5 f(x)\,dx$ = = $\frac{3x^3}{3}$ = x^3.
  - Apply the limits of integration: $x^3 \,|_2^5$ = $(5)^3 - (2)^3$ = 125 – 8 = 117 units squared is the area.

2.6 Average Value of Function over Interval

- The average value of a function f over the interval [a, b] is given by the formula: $\frac{1}{b-a}\int_a^b f(x)dx.$
- This formula is **not** in your log tables so you need to learn it.
- The question may ask for average value of a function **or** average height of a function over an interval. Both forms mean the same.
- E.g. Calculate average height of the function f(x) = $3x^2$ over the interval (2,5).
  - From 2.5.2 $\int_2^5 3x^2\,dx$ = 117.
  - Average value over [2,5] = $\frac{1}{5-2} * 117$ = 39.