Trigonometry 2 – Solutions

Trigonometric formulae

Question 1.

We have a mixture of functions of A and 7A on the left side of this equation, which makes it difficult to start solving this equation. We will have to use the formulae for cos (A±B) and sin (A±B) to simplify it. If we let x = 4A and y = 3A then we can express 7A and A in terms of these:

$7A = x + y$

$A = x - y$

$\Rightarrow \quad (\cos 7A + \cos A) / (\sin 7A - \sin A)$

$= (\cos(x+y) + \cos(x-y)) / (\sin(x+y) - \sin(x-y))$

$= ([\cos x \cos y - \sin x \sin y] + [\cos x \cos y + \sin x \sin y])$

$\quad / ([\sin x \cos y + \cos x \sin y] - [\sin x \cos y - \cos x \sin y])$

$= (2\cos x \cos y) / (2 \cos x \sin y)$

$= (\cos y) / (\sin y)$

$= 1 / \tan y$

$= \cot y$

$= \cot 3A$

Radians

Question 2.

Each runner runs a part of a circle (an arc) with angle θ. Let r be the radius (in metres) of the circle Kate runs on. Then the length of the arc that Kate runs is $r\theta$ metres.

Since the lanes are both 1.2 m wide, the midpoints of the lanes must be 1.2 m apart, so the radius of the circle Helen runs must be (r + 1.2) m. Then the length of the arc that Helen runs is $(r+1.2)\theta$ metres.

$\Rightarrow$ Helen runs 1.2θ metres further than Kate.

$\Rightarrow 1.2\theta = 3$

$\Rightarrow \theta = 2.5$ rad

Trigonometry 2 – Solutions

Solving equations

Question 3.

Let $x = \cos\theta$

We can use the triginometric formulae to express $\cos 2\theta$ in terms of $\cos\theta$ and $\sin\theta$, and then eliminate $\sin\theta$ so we express it purely in terms of $\cos\theta$. (Recall that $\cos^2\theta$ just means the square of $\cos\theta$.)

$\cos 2\theta = \cos^2\theta - \sin^2\theta = \cos^2\theta - (1 - \cos^2\theta) = 2\cos^2\theta - 1 = 2x^2 - 1$

$\Rightarrow \quad 2x^2 - 1 = 1/9$

$\Rightarrow \quad 2x^2 = 10/9$

$\Rightarrow \quad x^2 = 5/9$

$\Rightarrow \quad x = \pm\sqrt{5}/3$

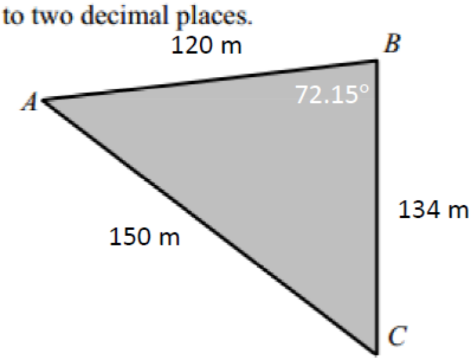
ANSWER: $\cos\theta = \pm\sqrt{5}/3$

Trigonometry 2 – Solutions

Sine rule, cosine rule, area of a triangle

Question 4.

(a) (i) Find $|\angle CBA|$. Give your answer, in degrees, correct to two decimal places.



$$\cos B = \frac{a^2+c^2-b^2}{2ac}$$
$$= \frac{120^2+134^2-150^2}{2(120)(134)}$$
$$= \frac{9856}{32160}$$
$$= 0 \cdot 306468$$
$$\Rightarrow B = 72 \cdot 15°$$

(ii) Find the area of the triangle *ACB* correct to the nearest whole number.

$$\text{Area } \Delta ABC = \tfrac{1}{2}ac\sin B = \tfrac{1}{2}(120)(134)\sin 72 \cdot 15$$
$$= 7652 \cdot 97$$
$$\approx 7653\,\text{m}^2$$

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Question 5.

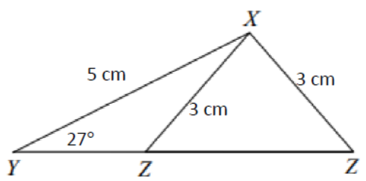
(i) Find the two possible values of $|\angle XZY|$. Give your answers correct to the nearest degree.

$$\frac{3}{\sin 27^\circ} = \frac{5}{\sin \angle Z} \Rightarrow \sin \angle Z = \frac{5 \sin 27^\circ}{3} = 0 \cdot 756$$

$$\Rightarrow |\angle Z| = 49^\circ \quad \text{or} \quad |\angle Z| = 131^\circ$$

- **Sin^{-1} (0.756) = 49.11°**
- **Sin is positive in the 1st and 2nd quadrants**
- **180° – 49° = 131°**

(ii) Draw a sketch of the triangle *XYZ*, showing the two possible positions of the point *Z*.



In the case that $|\angle XZY| < 90^\circ$, write down $|\angle ZXY|$, and hence find the area of the triangle *XYZ*, correct to the nearest integer.

$$|\angle ZXY| = 180^\circ - (27^\circ + 49^\circ) = 104^\circ$$

$$\Delta = \tfrac{1}{2} ab \sin C = \tfrac{1}{2}(5)(3)\sin 104^\circ = 7 \cdot 27 = 7 \text{ cm}^2$$

- Use 49° as 49°< 90°
- 27° is the angle at Y
- All angles must sum to 180° so the remaining angle must equal 104°

Trigonometry 2 – Solutions

Longer questions

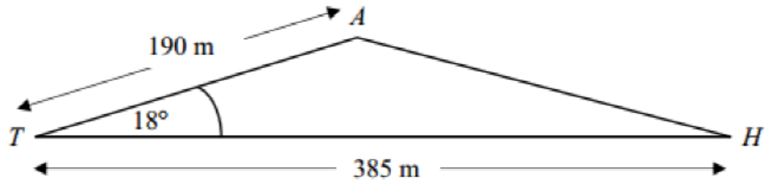
Question 6.

(a)

$$\sin \tfrac{1}{2}\alpha = \frac{15}{150} = 0\cdot 1$$
$$\Rightarrow \tfrac{1}{2}\alpha = 5\cdot 739°$$
$$\Rightarrow \alpha = 11\cdot 478°$$
$$\alpha = 11\cdot 5°$$

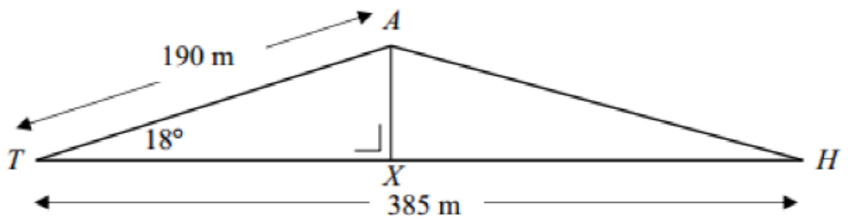
$\sin^{-1} 0.1 = 5.739°$

(b)



$$|AH|^2 = 190^2 + 385^2 - 2(190)(385)\cos 18°$$
$$= 36100 + 148225 - 139139\cdot 5683$$
$$= 45185\cdot 4317$$
$$|AH| = 212\cdot 57 = 213$$

Part (b) – alternative solution by splitting triangle into two right triangles:



Draw AX perpendicular to TH

triangle ATX: $\sin 18° = \frac{|AX|}{190} \Rightarrow |AX| = 58\cdot 71$

$$\cos 18° = \frac{|TX|}{190} \Rightarrow |TX| = 180\cdot 7$$
$$\Rightarrow |XH| = 204\cdot 3$$
$$\Rightarrow |AH|^2 = (58\cdot 71)^2 + (204\cdot 3)^2$$
$$\Rightarrow |AH| = 212\cdot 566 = 213$$

Trigonometry 2 – Solutions

(c)

(i)

The ball is at K at time t=0 so the height of K is h(0) = 8.

Answer: 8 m

(ii)

The ball reaches the point B when its height is zero. This occurs at a time t that satisfies h(t)=0.

$-6t^2 + 22t + 8 = 0$

Solve this using the quadratic formula. The solutions are 4 and -1/3. We take the positive solution t=4.

The horizontal speed is 38 m/s.

The travel time is 4 s.

$\Rightarrow$ The horizontal distance is 38 x 4 = 152 m.

Let α be the angle of elevation. Then tan (α) = 8 / 152.

$\tan^{-1}(8 / 152) = 3.01°$

Answer: 3°

(d)

(i)

$\tan(\theta) = ½ \Rightarrow h/d = ½$

$\Rightarrow d = 2h$

From the diagram we can see that:

|CD| = 25 – h

(ii)

We have a right triangle GDC with sides of length 2h, (25-h) and 25 (the hypotenuse). By Pythagoras:

$25^2 = (2h)^2 + (25 – h)^2$

$= 4h^2 + 25^2 + h^2 – 50h$

$\Rightarrow 5h^2 = 50h$

$\Rightarrow h = 10$

Question 7.

(a) (i)	$|EC|^2 = 3^2 + 2{\cdot}5^2 = 15{\cdot}25$ $|EC| = \sqrt{15{\cdot}25}$ $|EC| = 3{\cdot}905$ $\Rightarrow |AC| = 1{\cdot}9525$ $= 1{\cdot}95$
(a) (ii)	$\tan 50° = \frac{|AB|}{1{\cdot}95}$ $|AB| = 1{\cdot}95(1{\cdot}19175) = 2{\cdot}23239$ $|AB| = 2{\cdot}3$

Trigonometry 2 – Solutions

(a) (iii)	$\lvert BC\rvert^2 = 1{\cdot}95^2 + 2{\cdot}3^2$ $\lvert BC\rvert = 3\cdot 015377$ $\lvert BC\rvert = 3$ Also: $\sin 40° = \dfrac{1{\cdot}95}{\lvert BC\rvert}$ **or** $\cos 40° = \dfrac{2{\cdot}3}{\lvert BC\rvert}$ **or** $\cos 50° = \dfrac{1{\cdot}95}{\lvert BC\rvert}$ **or** $\sin 50° = \dfrac{2{\cdot}3}{\lvert BC\rvert}$
(a) (iv)	$3^2 = 3^2 + 2{\cdot}5^2 - 2(3)(2{\cdot}5)\cos\propto$ $15\cos\propto = 6{\cdot}25$ $\propto = 65°$ **or** $\cos\propto = \dfrac{1{\cdot}25}{3}$ $\propto = 65°$

Trigonometry 2 – Solutions

(a)

(v) $A = 2 \times \text{isosceles triangle} + 2 \times \text{equilateral triangle}$

$$= 2\times\left[\tfrac{1}{2}(2{\cdot}5)(3)\sin 65°\right] +$$

$$2 \times \left[\frac{1}{2}(3)(3)\sin 60°\right]$$

$$= 14{\cdot}59$$

$$A = 15$$

(b)

$$\tan 60° = \frac{3}{|CA|}$$

$$\Longrightarrow |CA| = \sqrt{3}$$

$$|CE| = 2\sqrt{3}$$

$$x^2 + x^2 = \left(2\sqrt{3}\right)^2$$

$$x = \sqrt{6}$$