Society of Actuaries in Ireland

Q1

a) Express $\frac{2\pi}{5}$ radians in degrees.

$$2\pi\ rads = 360° \ \ \ldots.\ divide\ both\ sides\ by\ 5$$

$$\frac{2\pi}{5} rads = \frac{360°}{5} = 72°$$

b) Express 210° in radians.

$$360° = 2\pi\ rads \ \ldots.\ divide\ both\ sides\ by\ 360$$

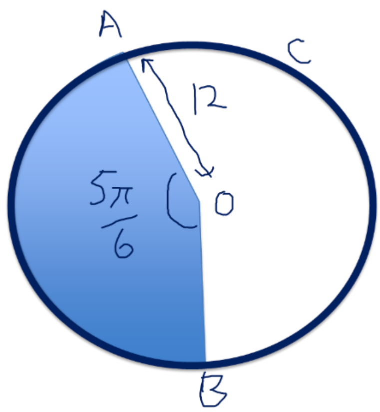
$$1° = \frac{2\pi}{360} rads = \frac{\pi}{180} rads \ \ \ldots multiply\ both\ sides\ by\ 210$$

$$210° = \frac{210\pi}{180} rads = \frac{7\pi}{6} rads \approx 3.67\ rads$$

Q2

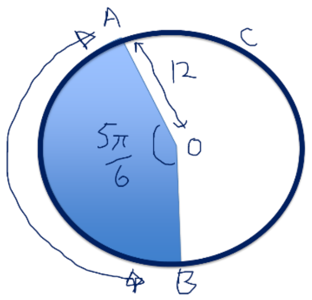
The diagram shows a circle c with centre O and radius 12cm. Also shown is the minor sector ABO. The minor arc [AB] subtends an angle of $\frac{5\pi}{6}$ rads at the centre.

i (i) Label the diagram.



ii (ii) Find the length of the minor arc [AB]

$$l = r\theta$$
$$= 12 \times \frac{5\pi}{6}$$
$$= 10\pi \text{ cm}$$



(ii) Find the area of the major sector ABO

$$A = \frac{1}{2} r^2 \theta$$
$$\theta = 2\pi - \frac{5\pi}{6} = \frac{7\pi}{6}$$
$$A = \frac{1}{2} \times 12^2 \times \frac{7\pi}{6} = 84\pi \text{ cm}^2$$

Q3

The diagram shows a triangle ABC.

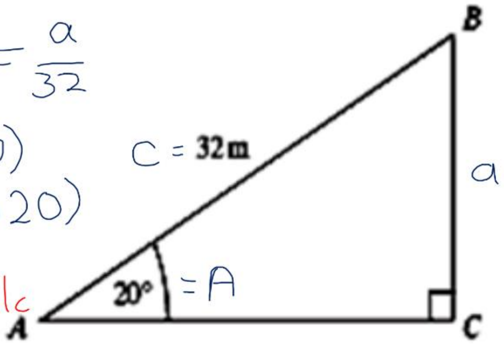
Angle A = 20° and angle C = 90° AB = 32m

Calculate the height |BC|.

Solve the triangle.

$$\text{Sin}(A) = \frac{a}{c} = \frac{a}{32}$$
$$a = 32 \,\text{Sin}(20)$$
$$a = 32\,(0{\cdot}342020)$$
$$a = 10{\cdot}9446 \text{ m}$$

degree mode on calc



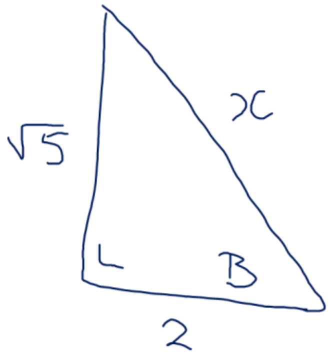
You can then solve for |AC| using the Pythagoras Theorem.

And the sum of the three angels in a triangle = 180, so:

<ABC = 180 – 90 – 20 = 70 degrees

Q4

If $\tan B = \sqrt{5} / 2$, find the value of $\sin B$ and $\cos B$.



$$x^2 = (\sqrt{5})^2 + 2^2$$
$$= 5 + 4 = 9$$
$$x = \sqrt{9} = 3$$

$$\sin B = \frac{\sqrt{5}}{3} \qquad \cos B = \frac{2}{3}$$

Q5

1) Find $\cos 72°18'$, correct to 4 decimal places.

<u>Method 1</u>

calculator:

$\boxed{\cos}$ 72 $\boxed{\circ ' ''}$ 18 $\boxed{\circ ' ''}$ $\boxed{=}$

$$= 0.30403306$$
$$= 0.3040 \text{ to 4 d.p}$$

<u>Method 2</u>

$$72 + \frac{18}{60} = 72.3$$

$$\cos(72.3) = 0.30403306$$
$$= 0.3040 \text{ to 4 dp}$$

2) If $\sin A = 0.5216$, find A correct to the nearest second.

<u>Method 1</u>

$\sin^{-1}(\sin A) = \sin^{-1}(0.5216)$

$A = \boxed{\text{Shift}}\ \boxed{\sin}\ 0.5216$

$= 31.43963765°$

$\boxed{°\ '\ ''} = 31° 26' 22.7''$

$= 31° 26' 23''$ to nearest second

<u>Method 2</u>

$31.43963765°$

$= 31° + 0.43963765 \times 60'$

$= 31° + 26.37825916'$

$= 31° + 26' + 0.37825916 \times 60''$

$= 31° 26' 22.6955'' = 31° 26' 23''$ to nearest sec

3) If $\sin A = 4/7$, find A

$\sin^{-1}(\sin A) = \sin^{-1}\left(\frac{4}{7}\right)$ ✓

$A = \boxed{\text{Shift}}\ \boxed{\sin}\ \boxed{(}\ 4\ \boxed{\div}\ 7\ \boxed{)}\ \boxed{=}$ ✓

$A = \boxed{\text{Shift}}\ \boxed{\sin}\ 4\ \boxed{\div}\ 7\ \boxed{=}$ ✗

$A = 34.8°$

4) Given $D = 3/4\pi$ $Rads$ find cosec D

Cosec is the reciprocal of Sine

$\text{Cosec}\left(\frac{3\pi}{4}\right) = \left(\sin\left(\frac{3\pi}{4}\right)\right)^{-1} = \frac{1}{\sin\left(\frac{3\pi}{4}\right)}$

Radian Mode

$= 1.414213562$

$\boxed{(}\ \sin\ \boxed{(}\ 3\ \boxed{\pi}\ \boxed{\div}\ 4\ \boxed{)}\ \boxed{)}\ \boxed{x^{-1}}\ \boxed{=}$

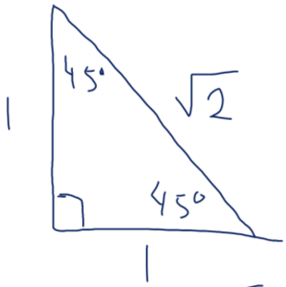
Q6

Make sketches of the following triangles:

• An Isosceles right-angled triangle with sides = 1 unit.

• An Equilateral triangle with sides = 2 units. Draw a line to divide this triangle into two equal right-angled triangles.

Solve all three triangles and hence calculate Sin, Cos and Tan of 30°, 45° and 60° in surd form.



Pythagoras

$$x^2 = 1^2 + 1^2 = 2$$

$$x = \sqrt{2}$$

$$\theta + \theta + 90° = 180°$$

$$\theta = 45°$$

$\sin 45° = \frac{1}{\sqrt{2}}$ $\quad$ $\tan 45° = \frac{1}{1} = 1$

$\cos 45° = \frac{1}{\sqrt{2}}$

$$60° \div 2 = 30°$$

$$y^2 + 1^2 = 2^2$$

$$y^2 = 4 - 1 = 3$$

$$y = \sqrt{3}$$

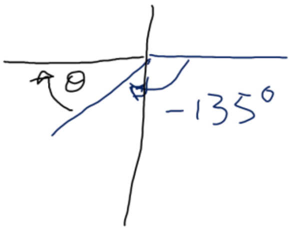
$\sin 60° = \frac{\sqrt{3}}{2}$ $\quad$ $\tan 60° = \frac{\sqrt{3}}{1} = \sqrt{3}$ $\quad$ $\sin 30° = \frac{1}{2}$

$\cos 60° = \frac{1}{2}$ $\quad$ $\tan 30° = \frac{1}{\sqrt{3}}$ $\quad$ $\cos 30° = \frac{\sqrt{3}}{2}$

Q7

1) Express in surd form, $cos\ (-135°)$.

Negative



$\theta = 180 - 135 = 45°$

3rd quad $\Rightarrow$ Cos is negative

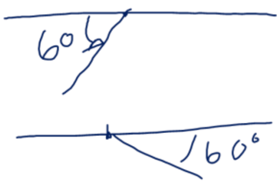
$\cos(-135°) = -\cos 45° = \frac{-1}{\sqrt{2}}$

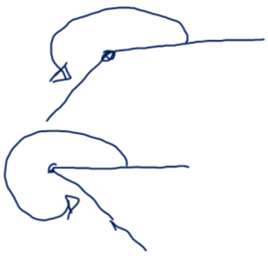
check on calculator but question asked for surd form.

2) If $\sin x = -\sqrt{3} / 2$, find two values for x if $0° \leq x \leq 360°$.

Answer is negative Sin is negative in 3rd + 4th Q

$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = 60°$ (pg 13 of formulae)





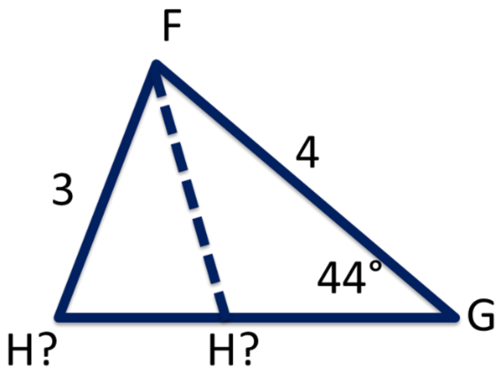
$180 + 60$
$= 240°$

$360 - 60$
$= 300°$

Q8

In a triangle FGH, $|FG|$ = 4cm, $|FH|$ = 3cm and $|\angle FGH| = 44°$.

Find the possible values of $\angle FHG$.



Use the Sine Rule

Pair? Yes = G and g. Second Pair? Have h but looking for H.

$$\frac{Sin\,H}{h} = \frac{Sin\,G}{g}$$

$$Sin\,H = h\frac{Sin\,G}{g}$$

$$H = \sin^{-1}\left(h\frac{Sin\,G}{g}\right)$$

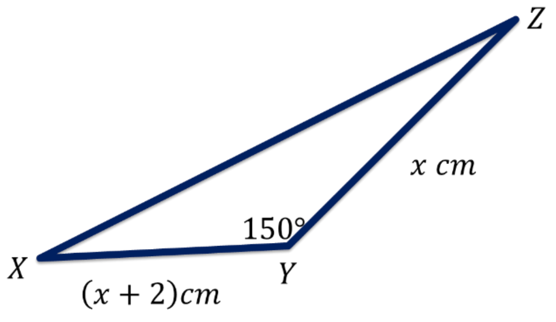
$$H = \sin^{-1}\left(4\frac{Sin\,44}{3}\right)$$

$$H = 67.851702°$$

or, $H = 180° - 67.85° = 112°$

Q9

Given that the area of this triangle is 6 cm2 find the value of x



$$Area = \frac{1}{2}xz\,Sin\,Y$$

$$6 = \frac{1}{2} x(x+2)\ Sin\ 150°$$

$$12 = x(x+2)\ Sin\ 30°$$

$$12 = x(x+2)\ \frac{1}{2}$$

$$24 = x(x+2)$$

$$x^2 + 2x - 24 = 0$$

$$(x+6)(x-4) = 0$$

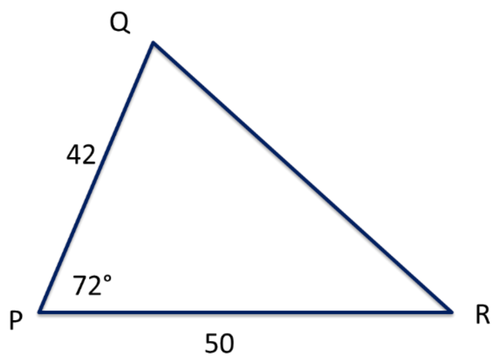
$$x = -6\ or\ x = 4$$

Can’t have a negative length,

therefore $x = 4$ cm

Q10

A builder ropes off a triangular plot of ground, PQR. The length of $|PQ| = 42\ m$ and the length of $|PR| = 50\ m$. $|\angle QPR| = 72°$. Calculate the length of rope needed by the builder. Give your answer correct to one decimal place.



Use the Cosine Rule

Pair? No SAS? Yes = rPq Looking for p

$$p^2 = r^2 + q^2 - 2rq \cos P$$

$$p^2 = 42^2 + 50^2 - 2(42)(50) \cos 72$$

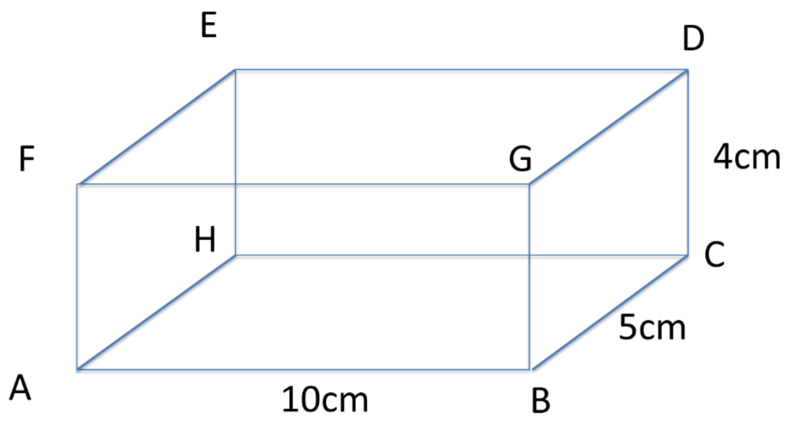
$$p^2 = 1764 + 2500 - (4200)(0.30901699) = 2966.1286236$$

$p = 54.4621760 = 54.5\ m$ correct to 1.d.p.

Rope needed = 42 + 50 + 54.5 = 146.5 m

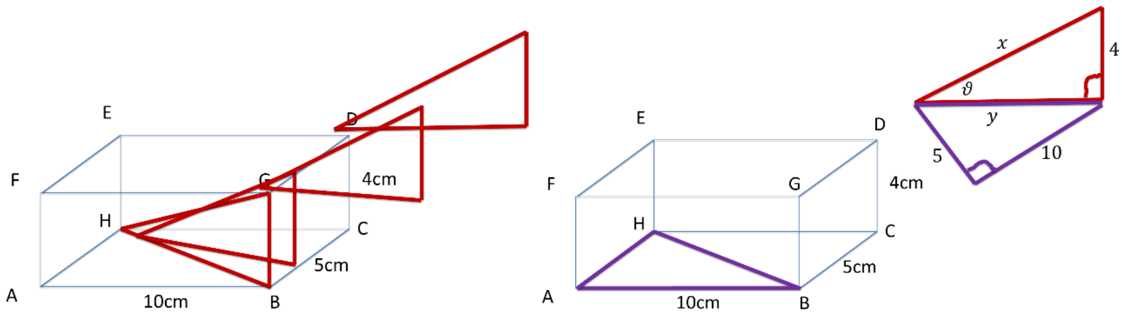
Q11

An open rectangular box has dimensions 10cm by 5cm by 4cm, as shown.



1) Find the length of the diagonal [GH].

2) Find the measure of the angle between GH and the base of the box.



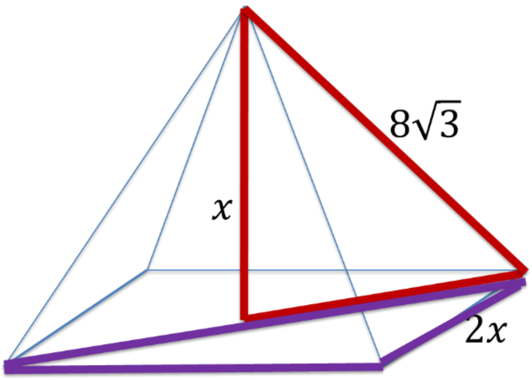
1) Use Pythagoras to find $y = \sqrt{125}$

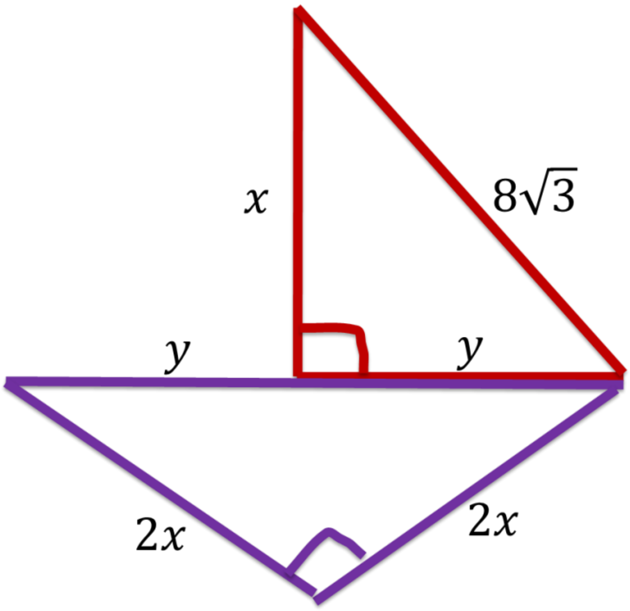
2) Use Pythagoras to find $\boxed{x = \sqrt{141}}$
$= 11.87434\text{cm}$

3) Use any ratio to find θ
eg $\theta = \text{Tan}^{-1}\left(\frac{4}{\sqrt{125}}\right) = \boxed{19.6857^\circ}$

Q12

The diagram represents a right pyramid. The base is a square of side $2x$ cm. The length of each of the slant edges is $8\sqrt{3}cm$. The height of the pyramid is $x\ cm$. Calculate the value of x.





$(2y)^2 = (2x)^2 + (2x^2)$

$y^2 = 2x^2$

$x^2 + y^2 = (8\sqrt{3})^2$

$x^2 + 2x^2 = 64 \times 3$

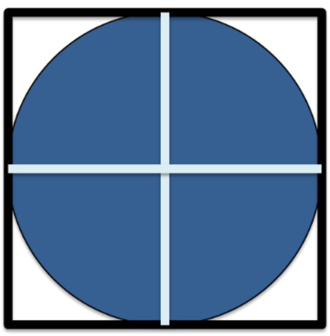
$3x^2 = 64 \times 3$

$x^2 = 64$

$x = 8$ cm

Q13

A square is inscribed in a circle, as shown. If the area of the circle is π square units, find the area of the square.



$A_c = \pi r^2 = \pi$

$\Rightarrow r = 1$

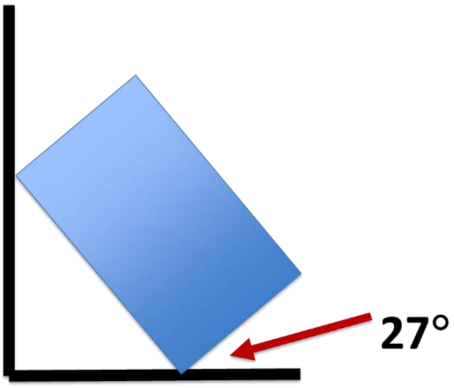
$A_s = 2r \times 2r$

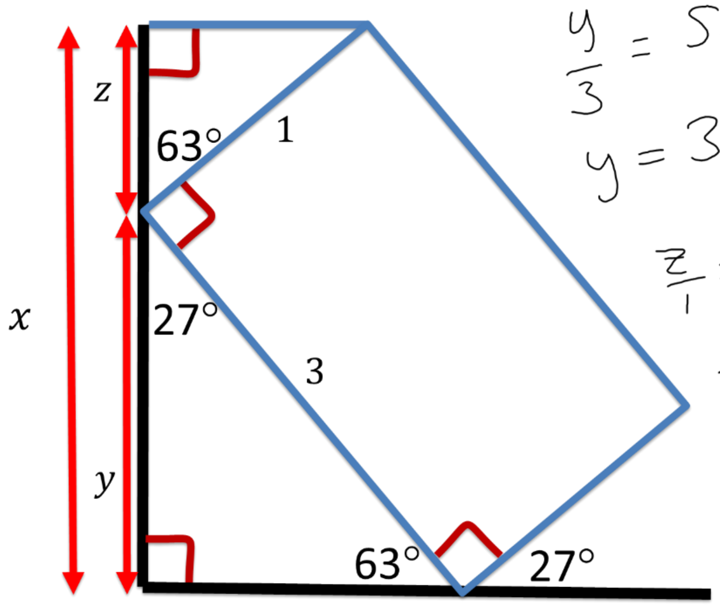
$= 4r^2$

$= 4$ sq units

Q14

A rectangular paving stone 3m by 1m rests against a vertical wall as shown. What is the height of the highest point of the stone above the ground? Give your answer in meters, correct to two decimal places.





$\frac{y}{3} = \sin 63°$

$y = 3 \sin 63°$

$\frac{z}{1} = \cos 63°$

$x = y + z$

$= 3 \sin 63° + \cos 63°$

$= 3.12701$

$= 3.13\text{m}$ to 2dp

Q15

Find all the solutions to the equation $\cos 3x = \sqrt{3}/2$, for $0° \leq x \leq 360°$.

$\cos 30° = \frac{\sqrt{3}}{2}$ from tables

Positive Cos in 1st and 4th quadrants

Two angles are $30°$ and $330°$

So $3x = 30°$ or $3x = 330°$

$x = 10°$ $\quad$ $x = 110°$

Period = 360/3 = 120

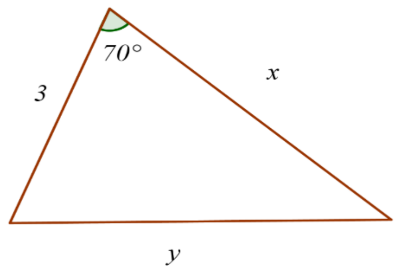
So, x=10+120=130, x=130+120=250, x=110+120=230, x=230+120=350

Q16

The area of the triangle shown is 15 square units.

– Find the value of x, correct to two decimal places.

– Using the Cosine Rule, find the value of y.



$$Area = \frac{1}{2}zxSinY = \frac{1}{2}(3)(x)Sin(70) = 1.409538931179x$$

$$Area = 15 = 1.409538931179x$$

$$x = \frac{15}{1.409538931179} = 10.64178 \approx 10.64\, units$$

$$y^2 = x^2 + z^2 - 2xzCosY$$

$$y^2 = x^2 + 3^2 - 2x(3)Cos70$$

$$y^2 = \left(\frac{15}{1.409538931179}\right)^2 + 3^2 - 2\left(\frac{15}{1.409538931179}\right)(3)Cos70$$

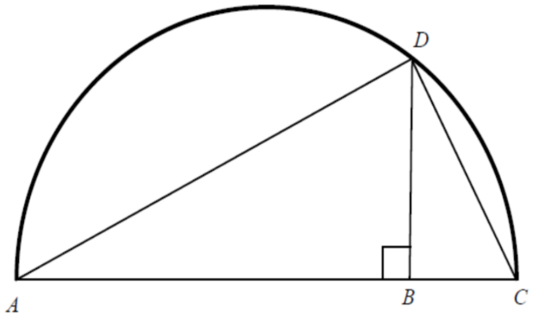
$$y^2 = 113.2474331 + 9 - 21.83821406$$

$$y^2 = 100.4092191$$

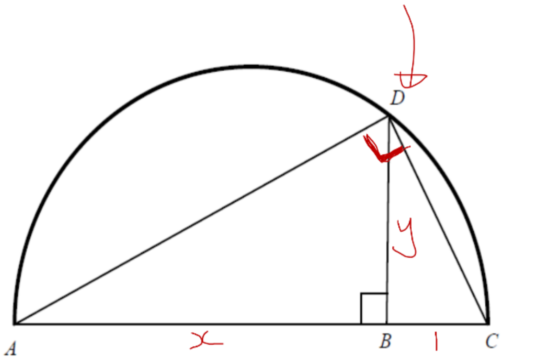
$$y = \sqrt{100.4092191} = 10.02044006 \approx 10.02$$

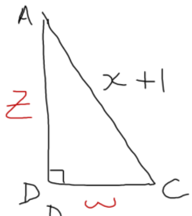
Q17

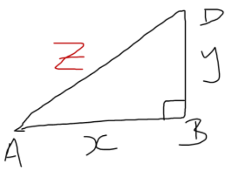
The diagram shows a semi-circle standing on a diameter $[AC]$, and $[BD] \perp [AC]$.
If $|AB| = x$ and $|BC| = 1$ and $|BD| = y$, write y in terms of x.



semi circle







$z^2 = \boxed{x^2 + y^2}$

$w^2 = (x+1)^2 - z^2 = \boxed{x^2 + 2x + 1 - z^2}$

$y^2 = w^2 - 1^2$ $\qquad$ $y^2 = x^2 + 2x - z^2$

$z^2 = x^2 + y^2$

$y^2 = x^2 + 2x - z^2$

$y^2 = x^2 + 2x - (x^2 + y^2)$

$y^2 = x^2 + 2x - x^2 - y^2$

$2y^2 = 2x$ $\qquad$ $y = \sqrt{x}$