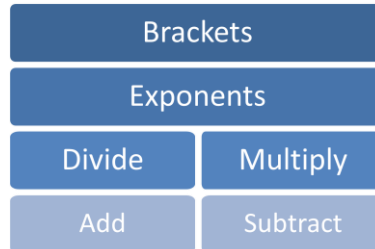




Simplifying an expression. What order should you follow?



To manipulate an equation, you can:

- Add or subtract the same number to/from both sides
- Multiply or divide both sides by the same number
- Square both sides, cube both sides, etc.
- Take the square root of both sides or cube root, etc

Simple Algebraic Rules

- Multiplying Terms
 $(a) \times (bc) = abc$
 $(a) \times (b+c) = ab + ac$
 $(a+b) \times (c+d) = a(c+d) + b(c+d)$

- Anything to the power of 0 is 1.

$$A^0 = 1, \quad 473^0 = 1, \quad \pi^0 = 1, \quad 1,000,000^0 = 1$$

- Inverse Powers

$$X^{-1} = \frac{1}{X}, \quad Y^{-6} = \frac{1}{Y^6}$$

- Indices

$$Y^3 + Y^2 = Y \times Y \times Y + Y \times Y \quad \dots \text{Can't be simplified}$$

$$(Y^2)^3 = (Y \times Y)^3 = (Y \times Y) \times (Y \times Y) \times (Y \times Y) = Y^6 \quad \dots 2 \times 3 = 6$$

$$Y^2 \times Y^3 = (Y \times Y) \times (Y \times Y \times Y) = Y^5 \quad \dots 2 + 3 = 5$$

$$\frac{Y^3}{Y^2} = \frac{Y \times Y \times Y}{Y \times Y} = Y^1 \quad \dots 3 - 2 = 1$$



Cancelling terms in fractions:

$$\frac{\cancel{A}}{A \times B} = \frac{1}{B} \quad \checkmark$$

$$\frac{X^2+2X}{X} = \frac{\cancel{X}(X+2)}{\cancel{X}} = \frac{X+2}{1} = X + 2 \quad \checkmark$$

$$\frac{\cancel{A}}{A + B} = \frac{1}{B} \quad \times$$

$$\frac{\cancel{X}+2}{\cancel{X}} = 2 \quad \times$$

Indices and Logarithms (From your Log Tables – Very Important!!!)

Indices	
$a^p a^q = a^{p+q}$	$a^{\frac{1}{q}} = \sqrt[q]{a}$
$\frac{a^p}{a^q} = a^{p-q}$	$a^{\frac{p}{q}} = \sqrt[q]{a^p} = (\sqrt[q]{a})^p$
$(a^p)^q = a^{pq}$	$(ab)^p = a^p b^p$
$a^0 = 1$	$\left(\frac{a}{b}\right)^p = \frac{a^p}{b^p}$
$a^{-p} = \frac{1}{a^p}$	

Logarithms	
General rule of logs: $a = b^c \iff \log_b a = c$	
$\log_a(xy) = \log_a x + \log_a y$	$\log_a\left(\frac{1}{x}\right) = -\log_a x$
$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$	$\log_a(a^x) = x$
$\log_a x^q = q \log_a x$	$a^{\log_a x} = x$
$\log_a 1 = 0$	$\log_b x = \frac{\log_a x}{\log_a b}$

Common mistake with Indices

$$(A + B)^2 = A^2 + B^2 \quad \times$$

$$\begin{aligned} (A + B)^2 &= (A + B) \times (A + B) \\ &= A \times (A + B) + B \times (A + B) \\ &= A^2 + AB + AB + B^2 \\ &= A^2 + 2AB + B^2 \quad \checkmark \end{aligned}$$



Algebra Hints and Tips

Common forms of **algebraic equations you might need to factorise:**

<p>Take out common terms</p> $ab + ad = a(b + d)$	<p>Factorise by grouping</p> $ab + ad + cb + cd = a(b + d) + c(b + d)$ $= (a + c)(b + d)$
<p>Factorise a trinomial</p> $a^2 - 2ab + b^2 = (a - b)(a - b) = (a - b)^2$	<p>Difference of two squares</p> $a^2 - b^2 = (a + b)(a - b)$
<p>Difference of two cubes</p> $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$	<p>Sum of two cubes</p> $a^3 + b^3 = (a + b)(a^2 - 2ab + b^2)$

Solving Quadratic Equations (i.e. finding the roots of a quadratic equation)

You can either:

- A. Factorise & let each factor = 0 ; OR
- B. Use the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$; OR
- C. Complete the square (write in vertex form) and set = 0

All three approaches will work. Some quadratics are easier to factorise than others, but you can always use the formula approach – even for simple quadratics.

Regarding the formula:

- If $b^2 - 4ac > 0$, the equation has two real distinct roots
- If $b^2 - 4ac = 0$, the equation has two equal real roots
- If $b^2 - 4ac < 0$, the equation has no real roots

Factor theorem

“If an algebraic expression is divided by one of its factors, then the remainder is zero. The expression $(x - k)$ is a factor of a polynomial $f(x)$ if the remainder when we divide $f(x)$ by $(x - k)$ is zero.”

- If $f(k) = 0$, then $(x - k)$ is a factor of $f(x)$
- If $(x - k)$ is a factor of $f(x)$, then $f(k) = 0$



Solving cubic equations ($ax^3 + bx^2 + cx + d = 0$)

- Find the first root, k , by trial and error
- If $x = k$ is a root, then $(x - k)$ is a factor
- Divide $f(x)$ by $(x - k)$, which always gives a quadratic expression
- Find the factors of the quadratic and then find the roots of the quadratic

Inequalities

Always remember that multiplying or dividing both sides of an inequality by a negative number reverses the direction of the inequality symbol

Quadratic inequalities

Replace $>$, $<$, \geq , \leq with $=$ to make an equation

- Solve the equation to find the roots \rightarrow these are called the critical values of the inequality & the solution either will be between these critical values or outside these critical values
- Test a number between the critical values (often 0) in the original inequality

Two possibilities arise:

- If the inequality is true, then the solution lies between the critical values
- If the inequality is false, then the solution does not lie between the critical values

(Note: that if the inequality uses $<$ or $>$, the critical values are not included in the solution set, whereas if the inequality uses \leq or \geq , the critical values are included in the solution set)

Modulus inequalities

If $|x| \leq a$, then $-a \leq x \leq a$

(Note: “mod x ” is the same as “ $|x|$ ”)

If $|x| \geq a$, then $x \leq -a$ or $x \geq a$



Surds

Properties of surds...

$\sqrt{ab} = \sqrt{a}\sqrt{b}$
$\sqrt{\left(\frac{a}{b}\right)} = \frac{\sqrt{a}}{\sqrt{b}}$
$\sqrt{a}\sqrt{a} = a$

To simplify surds...

Find the largest possible perfect square number greater than 1 that will divide evenly into the number under the square root.

Then use the first property above. E.g. $\sqrt{63} = \sqrt{(9 \times 7)} = \sqrt{9} \times \sqrt{7} = 3\sqrt{7}$



Lowest Common Denominator

When you want to sum (or subtract) two fractions, you need to find a common denominator. The easiest way is usually to multiply the two denominators (this will give you a common denominator, but it won't necessarily be the lowest one).

E.g.

$$\frac{1}{7} + \frac{3}{5}$$

Lowest Common denominator = $7 \times 5 = 35$

Now multiply so that you have the same denominator in each fraction:

$$\begin{aligned} \frac{5}{5}x\frac{1}{7} + \frac{3}{5}x\frac{7}{7} & \quad \text{(effectively multiplying by 1)} \\ = \frac{5}{35} + \frac{21}{35} & \quad \text{(you can combine once they have the same denominator)} \\ = \frac{26}{35} \end{aligned}$$

The same applies for algebraic fractions, e.g.

$$\frac{x}{y} + \frac{2x}{z}$$

Lowest Common denominator = yz

$$\begin{aligned} \frac{z}{z}x\frac{x}{y} + \frac{2x}{z}x\frac{y}{y} \\ = \frac{zx}{zy} + \frac{2xy}{zy} \\ = \frac{zx+2xy}{zy} \end{aligned}$$