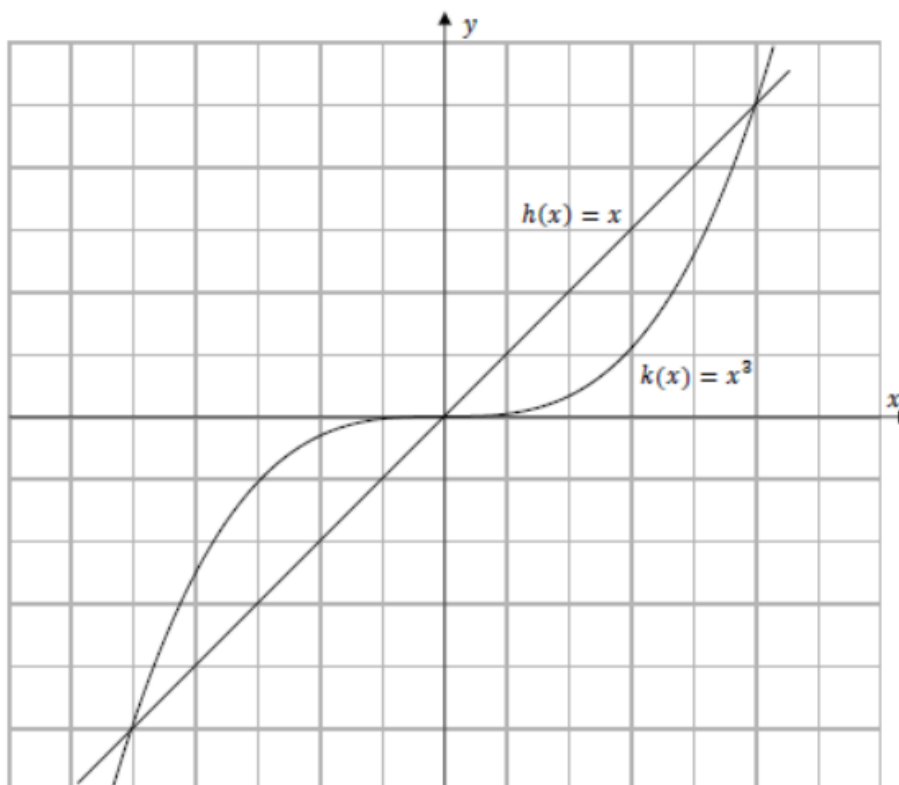


Calculus

Question 1. 2018 P1 Q6 (25 Marks)

Parts of the graphs of the functions $h(x) = x$ and $k(x) = x^3$, $x \in \mathbb{R}$, are shown in the diagram below.



- (a) Find the co-ordinates of the points of intersection of the graphs of the two functions.
- (b) (i) Find the total area enclosed between the graphs of the two functions.
- (ii) On the diagram on the previous page, using symmetry or otherwise, draw the graph of k^{-1} , the inverse function of k .

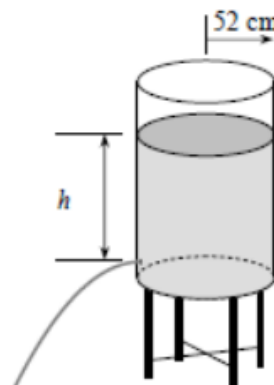
Question 2. 2012 P1 Q7 (50 Marks)

An open cylindrical tank of water has a hole near the bottom. The radius of the tank is 52 cm. The hole is a circle of radius 1 cm. The water level gradually drops as water escapes through the hole.

Over a certain 20-minute period, the height of the surface of the water is given by the formula

$$h = \left(10 - \frac{t}{200}\right)^2$$

where h is the height of the surface of the water, in cm, as measured from the centre of the hole,
and t is the time in seconds from a particular instant $t = 0$.



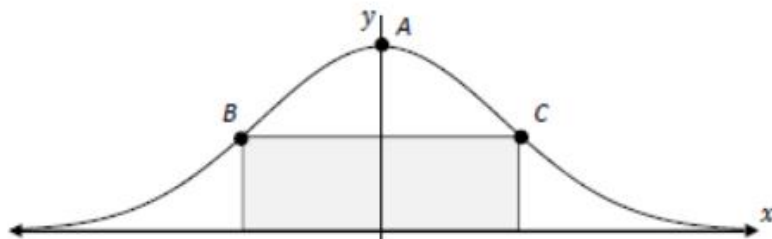
- What is the height of the surface at time $t = 0$?
- After how many seconds will the height of the surface be 64 cm?
- Find the rate at which the volume of water in the tank is decreasing at the instant when the height is 64 cm.
Give your answer correct to the nearest cm^3 per second.
- The rate at which the volume of water in the tank is decreasing is equal to the speed of the water coming out of the hole, multiplied by the area of the hole. Find the speed at which the water is coming out of the hole at the instant when the height is 64 cm.
- Show that, as t varies, the speed of the water coming out of the hole is a constant multiple of \sqrt{h} .
- The speed, in centimetres per second, of water coming out of a hole like this is known to be given by the formula

$$v = c\sqrt{1962h}$$

where c is a constant that depends on certain features of the hole.
Find, correct to one decimal place, the value of c for this hole.

Question 3. 2018 P1 Q8 (40 Marks)

The graph of the symmetric function $f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$ is shown below.



- (a) Find the co-ordinates of A, the point where the graph intersects the y-axis.
Give your answer in terms of π .

- (b) The co-ordinates of B are $\left(-1, \frac{1}{\sqrt{2\pi e}}\right)$. Find the area of the shaded rectangle in the diagram above. Give your answer correct to 3 decimal places.

- (c) Use calculus to show that $f(x)$ is decreasing at C.

- (d) Show that the graph of $f(x)$ has a point of inflection at B.

Probability

Question 4. 2012 P2 Q7 (d) (Part of a 75 mark question)

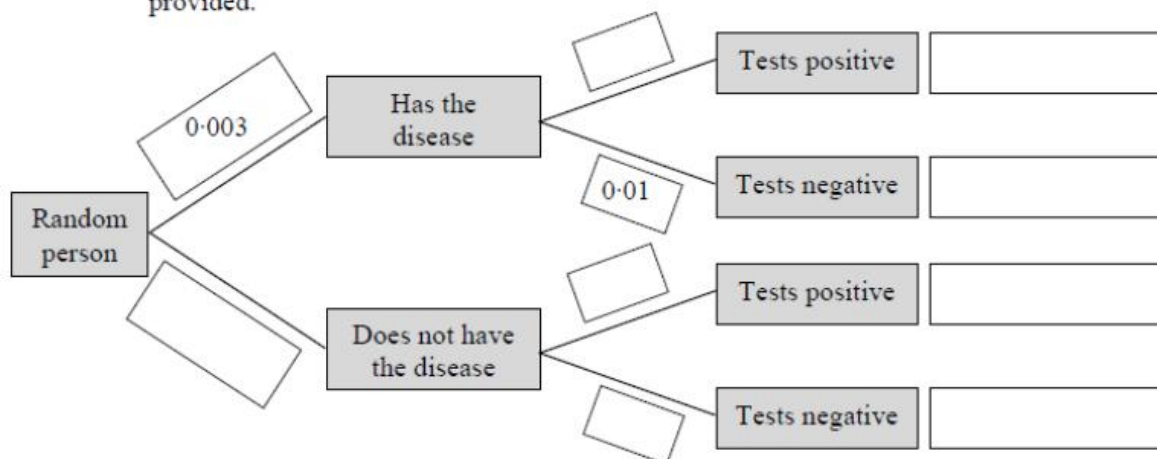
To buy a home, people usually take out loans called *mortgages*. If one of the repayments is not made on time, the mortgage is said to be *in arrears*. One way of considering how much difficulty the borrowers in a country are having with their mortgages is to look at the percentage of all mortgages that are in arrears for 90 days or more. For the rest of this question, the term *in arrears* means in arrears for 90 days or more.

- (d) A property is said to be in “negative equity” if the person owes more on the mortgage than the property is worth. A report about mortgaged properties in Ireland in December 2010 has the following information:
- Of the 475 136 properties examined, 145 414 of them were in negative equity.
 - Of the ones in negative equity, 11 644 were in arrears.
 - There were 317 355 properties that were neither in arrears nor in negative equity.
- (i) What is the probability that a property selected at random (from all those examined) will be in negative equity?
Give your answer correct to two decimal places.
- (ii) What is the probability that a property selected at random from all those in negative equity will also be in arrears?
Give your answer correct to two decimal places.
- (iii) Find the probability that a property selected at random from all those in arrears will also be in negative equity.
Give your answer correct to two decimal places.

Question 5. 2014 P2 Q8 (a) (Part of a 45 mark question)

Blood tests are sometimes used to indicate if a person has a particular disease. Sometimes such tests give an incorrect result, either indicating the person has the disease when they do not (called a false positive) or indicating that they do not have the disease when they do (called a false negative). It is estimated that 0.3% of a large population have a particular disease. A test developed to detect the disease gives a false positive in 4% of tests and a false negative in 1% of tests. A person picked at random is tested for the disease.

- (a) (i) Write the probability associated with each branch of the tree diagram in the blank boxes provided.

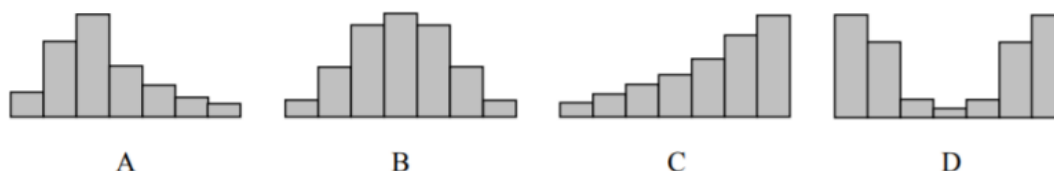


- (ii) Hence, or otherwise, calculate the probability that a person selected at random from the population tests positive for the disease.
-
- (iii) A person tests positive for the disease. What is the probability that the person actually has the disease? Give your answer correct to three significant figures.
- (iv) The health authority is considering using a test on the general population with a view to treatment of the disease. Based on your results, do you think that the above test would be an effective way to do this? Give a reason for your answer.

Statistics

Question 6. 2021 Sample SEC P2 Q2

The shapes of the histograms of four different sets of data are shown below.



- (a) Complete the table below, indicating whether the statement is correct (✓) or incorrect (✗) with respect to each data set.

	A	B	C	D
The data are skewed to the left				
The data are skewed to the right				
The mean is equal to the median				
The mean is greater than the median				
There is a single mode				

- (b) Assume that the four histograms are drawn on the same scale. State which of them has the largest standard deviation, and justify your answer.

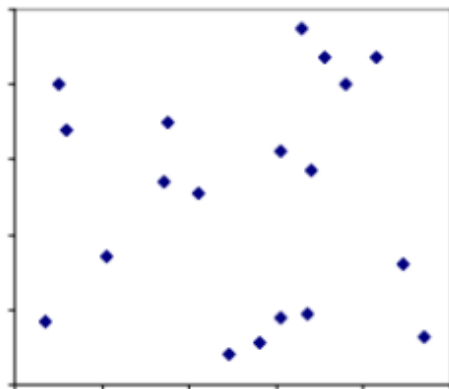
Question 7. 2012 P2 Q5 (25 marks)

A company produces calculator batteries. The diameter of the batteries is supposed to be 20 mm. The tolerance is 0.25 mm. Any batteries outside this tolerance are rejected. You may assume that this is the only reason for rejecting the batteries.

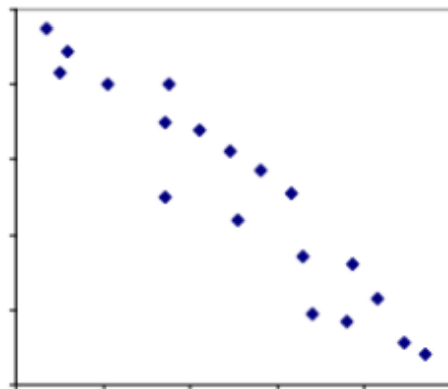
- The company has a machine that produces batteries with diameters that are normally distributed with mean 20 mm and standard deviation 0.1 mm. Out of every 10 000 batteries produced by this machine, how many, on average, are rejected?
- A setting on the machine slips, so that the mean diameter of the batteries increases to 20.05 mm, while the standard deviation remains unchanged. Find the percentage increase in the rejection rate for batteries from this machine.

Question 8. 2010 P2 Q3

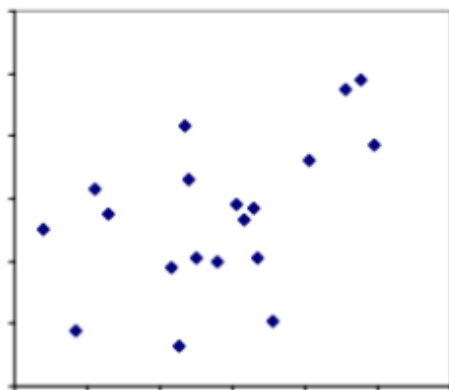
(a) For each of the four scatter plots below, estimate the correlation coefficient.



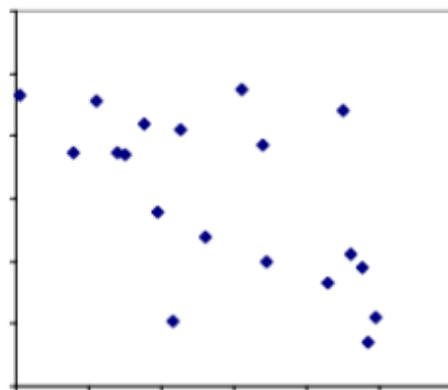
Correlation \approx _____



Correlation \approx _____



Correlation \approx _____



Correlation \approx _____

(b) Using your calculator, or otherwise, find the correlation coefficient for the data given in the table. Give your answer correct to two decimal places.

x	y
0.0	0.5
5.0	1.3
5.2	3.3
6.1	6.7
9.3	4.5
9.5	4.6
9.9	6.5

Financial Maths

Question 9. 2014 SEC Sample Paper 1 Q8 (50 marks)

Pádraig is 25 years old and is planning for his pension. He intends to retire in forty years' time, when he is 65. First, he calculates how much he wants to have in his pension fund when he retires. Then, he calculates how much he needs to invest in order to achieve this. He assumes that, in the long run, money can be invested at an inflation-adjusted annual rate of 3%. Your answers throughout this question should therefore be based on a 3% annual growth rate.

- (a) Write down the present value of a future payment of €20,000 in one year's time.
- (b) Write down, in terms of t , the present value of a future payment of €20,000 in t years' time.
- (c) Pádraig wants to have a fund that could, from the date of his retirement, give him a payment of €20,000 at the start of each year for 25 years. Show how to use the sum of a geometric series to calculate the value, on the date of retirement, of the final fund required.
- (d) Pádraig plans to invest a fixed amount of money every month in order to generate the fund calculated in part (c). His retirement is $40 \times 12 = 480$ months away.
 - a. Find, correct to four significant places, the rate of interest per month that would, if paid and compounded monthly, be equivalent to an effective annual rate of 3%.
 - b. Write down, in terms of n and P , the value on the retirement date of a payment of € P made n months before the retirement date.
 - c. If Pádraig makes 480 equal payments of € P from now until his retirement, what value of P will give him the fund he requires?
- (e) If Pádraig waits for ten years before starting his pension fund, how much will he then have to pay each month in order to generate the same pension fund?

Other Requested Topics

Question 10. Logs

Given that $p = \log_c x$, express $\log_c \sqrt{x} + \log_c (cx)$ in terms of p

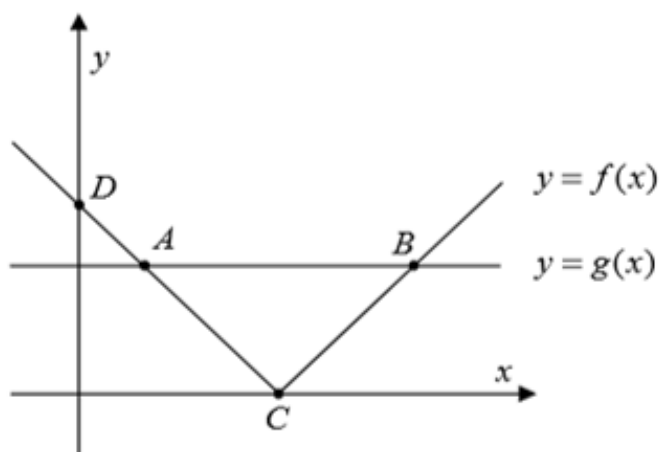
Question 11. Functions and Inequalities

a, Solve the simultaneous equations

$$2x + 8y - 3z = -1$$

$$2x - 3y + 2z = 2$$

$$2x + y + z = 5$$



b, The graphs of the functions $f: x \rightarrow |x-3|$ and $g: x \rightarrow 2$ are shown in the diagram.

- i) Find the co-ordinates of the points A, B, C and D.
- ii) Hence, or otherwise, solve the inequality $|x-3| < 2$