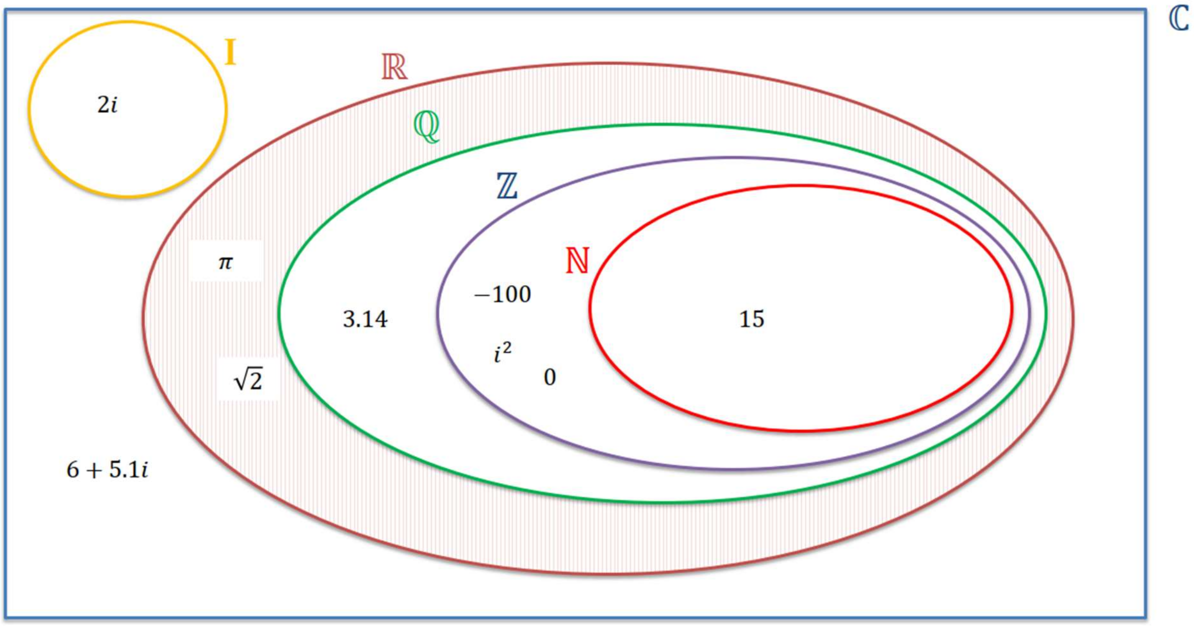
Society of Actuaries in Ireland

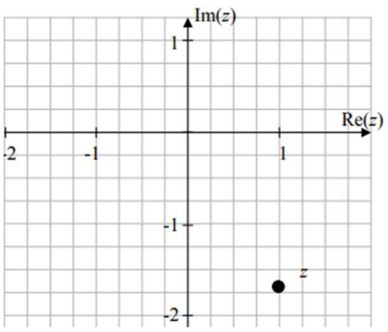
Hints and Tips – Complex Numbers & Proof by Induction

$i = \sqrt{(-1)}$



Where N is all Natural Numbers, Z is all Integers, Q is all Rational Numbers, R is all Real Numbers, I is all Imaginary Numbers and C is all Complex Numbers

Plotting a complex number on Argand diagram z = 1 - 1.8i



Conjugates

Conjugate of a+bi = $\overline{a+bi}$ = a-bi

$\overline{Z_1 . Z_2} = \overline{Z_1} . \overline{Z_2}$

$\overline{Z_1 + Z_2} = \overline{Z_1} + \overline{Z_2}$

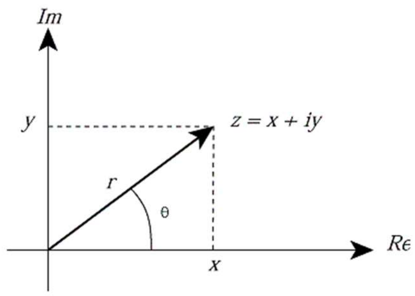
To divide by a complex number – multiply by 1 using conjugate above & below line i.e. :

$$\frac{a+bi}{e+fi} \Rightarrow \frac{a+bi}{e+fi} \times \frac{e-fi}{e-fi}$$

Conjugate is equivalent to reflection in the real axis.

<u>Modulus</u> is the distance of point from point 0 (i.e. from 0 + 0i) on Argand diagram

$|x+iy| = \sqrt{(x^2 + y^2)}$



Cos θ = x/r   Sin θ = y/r

Tan θ = y/x

$|z_1 z_2| = |z_1|.|z_2|$

<u>Polar Form</u>:

Polar Form of z, where z = x+iy is rCosθ+irSinθ where r = |x+iy| so r>=0

Using Cos θ = x/r   Sin θ = y/r

<u>De Moivre's Thoerom</u>

$[r\text{Cos}\theta+ir\text{Sin}\theta]^k = [r^k\text{Cos}(k\theta)+ir^k\text{Sin}(k\theta)] = r^k(\text{Cos}(k\theta)+\text{Sin}(k\theta))$

θ or angle/argument is measured in radians

To find k roots use using De Moivre's Theorem set:

$r^{1/k}(\text{Cos}(\theta/k +2n\pi/k)+\text{Sin}(\theta/k+2n\pi/k))$ for n =0, 1,…,k-1

Proof by Induction

Step 1: Show that the proposition is true for $n = 1$

(insert your workings to show outcome…..)

Hence, proposition is true for n=1

Step 2: Assume that the proposition is true for n=k

Step 3: *Prove* that the proposition is true for $n = k + 1$, given that it is true for $n = k$.

(insert your workings to show outcome for k+1 & how it relates to outcome for k…..)

∴ The proposition is true for $n = k + 1$, given that it is true for $n = k$.

Step 4: State that proposition is true for $n = 1$ and if the proposition is true for n=k, it will be true for n = k + 1, therefore by induction it is true for all $n \in \mathrm{N}$