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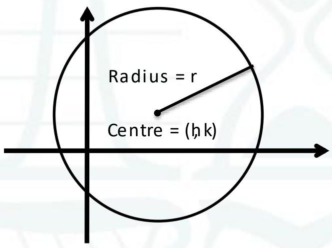
Coordinate Geometry: The Circle – Hints & Tips

General:

Circle

Equation of the Circle

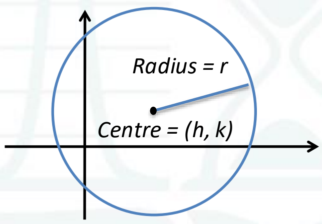


- The equation of a circle has an x^2 term and a y^2 term.
- Equation of a circle with centre (0, 0) and radius r: $x^2 + y^2 = r^2$.
- Equation of a circle with centre (h, k) and radius r: $(x - h)^2 + (y - k)^2 = r^2$ (Page 19)
- General equation of a circle: $x^2 + y^2 + 2gx + 2fy + c = 0$ (Page 19).
    - Centre = (-g, -f).
    - Radius = $\sqrt{g^2 + f^2 - c}$

When to use $(x - h)^2 + (y - k)^2 = r^2$:

Circle

When to use the equation $(x - h)^2 + (y - k)^2 = r^2$

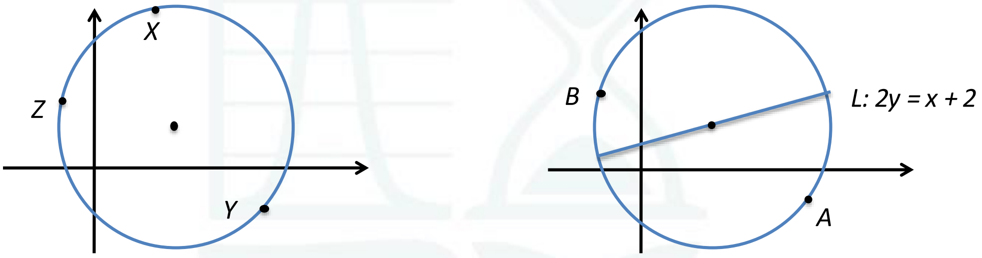


- If we know centre (h, k) and the radius use this equation.
- If we know the centre and a point on the circle:
    - Calculateradiusas distance between centre and the point on the circle.
- If we know the centre of the circle, it's easier to use the equation $(x - h)^2 + (y - k)^2 = r^2$

When to use $x^2 + y^2 + 2gx + 2fy + c = 0$:

Circle

When to use the equation $x^2 + y^2 + 2gx + 2fy + c = 0$

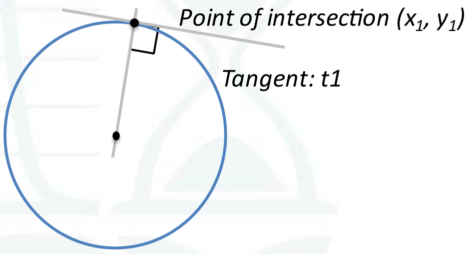


- If we have 3 points on the circle:
  - Substitute each point into the equation of the circle.
  - This will give 3 equations with 3 unknowns (g, f and c). Solve simultaneous equations.
- If we have 2 points on the circle and equation of a line containing centre of circle:
  - Substitute each point into the equation of the circle.
  - Substitute (-g, -f) into the equation of the line.
  - This will given 3 equations with 3 unknowns (g, f and c). Solve simultaneous equations.
- There may be other scenariosgiven but if you can either get 3 points on the circleor 2 points on the circle and the equation of a line with the centre, then you can use the formula $x^2 + y^2 + 2gx + 2fy + c = 0$.

Tangents:

Circle

Tangents



- A tangent is a line which touches the circle at exactly one point.
- The slope of the tangent and the slope of the radius at the point of intersection are perpendicular.
- If we have the equation of a tangent and the point of intersection then we can find an equation of a line containing the centre of the circle.

- If a circle has the x axis as a tangent then: $g^2 = c$
- If a circle has the y axis as a tangent then: $f^2 = c$
- If a circle has the x axis and y axis as tangents then:
  - $g^2 = f^2 = c$
  - $g = +/- f$

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Circles Touching:

Circle

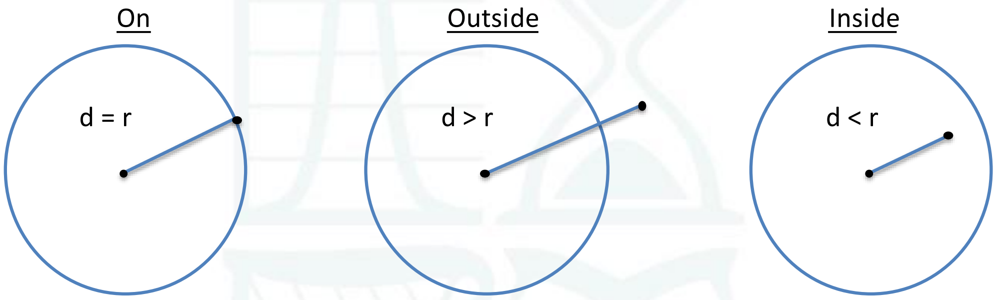
Circles Touching

- If two circles touch externally, the distance between the centres is the sum of their radii i.e. r1 + r2 = d
- If two circles touch internally, the distance between the centres is the difference of their radii i.e. r1 - r2 = d

Points on, outside or inside a circle:

Circle

Points on, outside or inside circle



- If a point is **on** the circle: distance from the centre to the point **is equal** to the radius
- If a point is **inside** the circle: distance from the centre to the point **is less** to the radius
- If a point is **outside** the circle: distance from the centre to the point **is greater** to the radius

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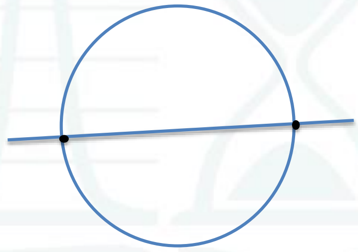
Point(s) of Intersection between Line and Circle:

Circle

Finding point(s) of intersection between line and circle

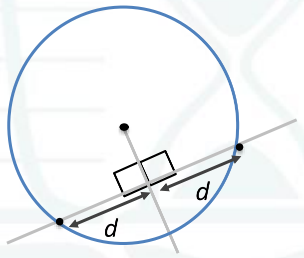


- If we know the equation of the circle and know the equation of the line that intersects the circle:
  - Get x in terms of y from the equation of the line, or y in terms of x. E.g. If the equation of the line is $2y = x + 3$. Then $x = 2y - 3$.
  - Substitute this new term for x into the equation of the line. E.g. If the equation of the circle is $(x-3)^2 + (y-6)^2 = 36$ the substitute $(2y - 3)$ for each x term in the equation of the circle.
  - Solve the quadratic equation to find the y coordinates at the points of intersection.
  - Sub back in the values of y from above into the equation of the line to find the x coordinates.
  - If the line is a tangent then there will only be one point of intersection

Chords:

Circle

Perpendicular bisector of a chord



- A chord is a line joining two points on the circumference of a circle.
- A diameter is a type of chord.
- The perpendicular bisector of a chord is a line containing the centre of the circle.
- A bisector divides the chord into two equal parts.

Common Chord/ Tangent

- If two circles have the same chord/tangent in common then to find the equation of the tangent:
  - Use the equation $C_1 - C_2 = 0$ where C_1 and C_2 are the equations of the circles in the form $x^2 + y^2 + 2gx + 2fy + c = 0$
  - You can only use this if the x^2 and y^2 terms have the same coefficient for both circles. (i.e. when you subtract $C_1 - C_2$, the x^2 and y^2 terms cancel out.