Prudent Harmonized Reduced Set of Scenarios (PHRSS)
Methodological options for scenarios generation

Workshop with Stakeholders

15 September 2022

European Insurance and Occupational Pensions Authority

Content

- **Introduction – overview of the project**
- Methodological options to produce PHRSS
- General features of scenarios – discussion points

Overview of the project

Issue : EIOPA has identified discrepancies in the use of stochastic methods for the valuation of options and guarantees.

However to determine whether using deterministic methods is an immaterial simplification, a full ESG should be used in input of an ALM model designed to process a thousand of stochastic simulations (instead of one of a deterministic model).

The Objective of the PHRSS is therefore to provide a practical simplification to assess the materiality of options and guarantees only using (1) a set of scenario regularly provided by EIOPA (2) its current best estimate model, ran approximately 10 times to obtain de pseudo-stochastic Best Estimate.

Difference between a stochastic valuation and the PHRSS : The PHRSS is a materiality assessment tool that falls under the proportionality principle, and is therefore not expected to perfectly match the requirements of DR 22.3 regarding market consistency and martingale properties.

Objectives of the workshop

Share current EIOPA work on the methodology and the main milestones of the project

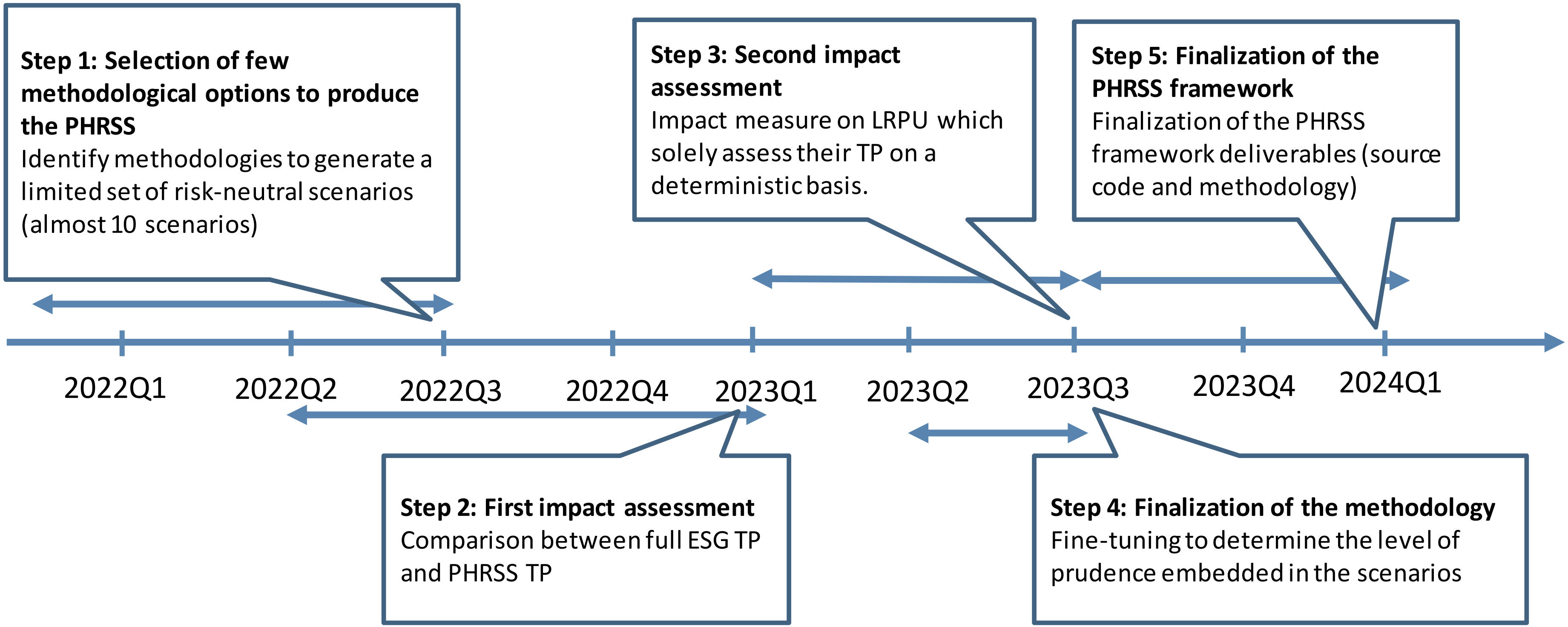
Give the opportunity to provide feedback regarding the methodological options

Exchange on practical questions regarding :

- **The risk factors to be included in the PHRSS**
- **The format of the tables**
- **The frequency of the publication**
- **…**

Introduction

Organization and timeline overview



First impact assessment

Objective: assess impacts of shortlisted PHRSS options on a sample of EEA undertakings that perform their TP assessment with full stochastic scenarios. The comparison between full ESG TP and PHRSS TP will allow improving the framework and prepare for a second impact assessment aimed on LRPU which solely assess their TP on a deterministic basis.

Process

- During Q4-2022:
  - Take into account industry's views on the template of scenarios
  - Select participating undertakings to the first impact assessment
- End of Q4-2022
  - Share final technical specifications with participating undertakings and launch impact assessment
  - EIOPA will publish the data request on its website.
  - NSAs will perform the communication with the participating undertakings and collect the data.
- Submission deadline : beginning of Q1-2023

Feedback: Undertakings will receive feedback on the results at the end of the study.

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Methodological options to produce PHRSS

Overview of PHRSS generation process

Objective: identification of methodological options allowing to generate a limited set of risk-neutral scenarios (almost 10 scenarios) and to assess a prudence level to reflect TVOG

A methodological option relies on the 3 underlying steps detailed below:

1 Generation of raw scenarios

Generation of an initial set of scenarios (almost 10 scenarios) by using risk-neutral models or real-world scenarios

Step requiring to design a methodology

2 Scenarios adjustments

Correction of raw scenarios to meet martingale and market consistency properties

Step requiring to design a methodology

3 Deterministic TP correction

- ✓ TVOG criterion: TVOG < 5% SCR ?
  - ✓ With TVOG = Stochastic (PHRSS) BE – Deterministic (single scenario) BE
- ✓ For LRPU which meet TVOG criterion: adjustment of deterministic BE (with a supplement equal to 5% of SCR or an ad-hoc(*) stochastic supplement using PHRSS)

Out of scope of slides

Question to stakeholders : does the 2 steps approach seems reasonable to you ?

(*) Subject to discussion with NCAs

Methodological options to produce PHRSS

Step 1: Generation of raw scenarios (1/4)

1 **Generation of raw scenarios**
Generation of an initial set of scenarios (almost 10 scenarios) by using risk-neutral models or real-world scenarios tailored by experts

2 **Scenarios adjustments**
Correction of raw scenarios to meet martingale and market consistency properties

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- ✓ TVOG criterion: TVOG < 5% SCR ?
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Principle: generation of an **initial set of scenarios** by using risk-neutral models or real-world scenarios tailored by experts

Candidate methodologies for raw scenarios generation:

- **Method 1.1 - Pure stochastic trajectories** : each simulation is **randomly generated** by a stochastic model
  - Calibrate a reference **risk-neutral / real-world ESG** with spot market data or prudent assumptions
  - Generate stochastically 10 scenarios
- **Method 1.2 - Percentiles level lines**: composed of **risk factors percentile** values
  - Calibrate a reference risk-neutral / real-world ESG with spot market data or prudent assumptions
  - Generate a full set of scenarios
  - Define **percentile trajectories** for almost **10 thresholds** (…,10%, 20%,…,50%,…,90%,…) by considering the **percentile values** of each risk factor over the projection horizon $\left(\left(q_\alpha(RF_i(t))\right)_{i,t}\right)_{\alpha=10\%,\ldots,90\%,\ldots}$
- **Remark:** economic scenarios might be generated by using a **real-world ESG** as they are adjusted in a second step to **meet martingale and market consistency properties**
  - In addition, underlying **volatilities** may be induced by replication of **Pillar 1 shocks** considered for **SCR Market** calculation (please refer to appendix for more details)

Methodological options to produce PHRSS

Step 1: Generation of raw scenarios (2/4)

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Generation of an initial set of scenarios (almost 10 scenarios) by using risk-neutral models or real-world scenarios tailored by experts

2 **Scenarios adjustments**
Correction of raw scenarios to meet martingale and market consistency properties

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✓ TVOG criterion: TVOG < 5% SCR ?
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Candidate methodologies for raw scenarios generation:

- **Method 1.3 - Ranked scenarios** : use of a **theoretical portfolio of assets** to measure the adversity level of each scenario and to rank them consequently
  - Calibrate a reference risk-neutral / real-world ESG with spot market data or prudent assumptions
  - Generate a full set of N scenarios
  - For each scenario, calculate the value V of the **reference portfolio** to obtain its **empirical distribution** $(V_s)_{s=1,\dots,N}$
  - Define **percentile trajectories** for almost 10 thresholds (…,10%, 20%,…,50%,…,90%,…) by considering the percentile values of $(V_s)_{s=1,\dots,N}$. For each threshold α, note s_α the scenario number which lead to α-percentile of V.
  - **Extract the trajectories** associated to the **simulations s_α** to get the set of raw scenarios: $\left(\left(RF_i^{s_\alpha}(t)\right)_{i,t}\right)_{\alpha=10\%,\dots,90\%,\dots}$
- **Focus point**: selected scenarios may strongly differ from an initial full set to another one
- **Remark 1**: the reference portfolio might be based on **assets EIOPA Insurance Statistics - Exposure data**.
  - The reference portfolio might be alternatively based on insurance industry liabilities cash-flows adjusted for each stochastic scenario (by taking into account profit sharing mechanisms and discounting) but it would potentially lead to less generic approach as liabilities specificities may strongly differ from a country to another one
- **Remark 2**: an **entity specific VA** calculation would lead to generate scenarios **per undertaking** (based on entity specific yield curves)
  - In such a case, PHRSS could be delivered for different buckets of VA and then adapted to each entity by some market consistent adjustments

Methodological options to produce PHRSS

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Candidate methodologies for raw scenarios generation:

- **Method 1.4 - Alternative approach :** consider **conditional expectations** of risk factors **given percentiles** of reference **portfolio**
  - This approach consists in defining percentile scenarios as follows $\left(\left(E(RF_i(t)|V = q_\alpha(V))\right)_{i,t}\right)_{\alpha=10\%,\ldots,90\%,\ldots}$
  - Such elements may be evaluated **theoretically by closed-formula** OR by **numerical techniques**
    - If N_α denotes the neighbourhood of percentile scenario s_α, $E(RF_i(t)|V = q_\alpha(V))$ may be estimated by $\overline{RF}_i^\alpha(t) = \frac{1}{\#N_\alpha}\sum_{s\in N_\alpha} RF_{i,s}(t)$
- **Method 1.5 - Real-world scenarios** defined **"by hand"** by experts
  - Build the set of raw scenarios by using some **"by hand" experts anticipations** based on historical financial data series

Methodological options to produce PHRSS

Step 1: Generation of raw scenarios (4/4)

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Below a summary of pros and cons per option:

Generation of raw scenarios	Pros	Cons	Score
Method 1.1 - Pure stochastic trajectories	✓ No specific treatments expected. Only requires standard risk-neutral simulations.	✓ Scenarios may strongly differ from a set to another one (high dependency to the seed of random number generator)	
Method 1.2 - Percentiles level lines	✓ Stability of the method (no dependency to the seed if percentiles are estimated on a sufficient number of scenarios)	✓ Trajectories may be too adverse or favorable for some thresholds (due to limited mitigation effects over time horizon) => need to adjust adequately level of thresholds	
Method 1.3 - Ranked scenarios	✓ Adversity level of trajectories more coherent (approach allowing to reflect mitigation effects over time horizon and between risks)	✓ Scenarios may strongly differ from a set to another one (high dependency to the seed of random number generator)	
Method 1.4 - Ranked scenarios with conditional expectations	✓ Adversity level of trajectories more coherent (mitigation effects over time and between risks) ✓ Stability of the method (no strong dependency to the seed)	✓ Trajectories may be no fully intuitive (as it is often the case with allocation techniques of diversification effects)	
Method 1.5 - Real-world scenarios defined "by hand" by experts	✓ Trajectories quite intuitive (based usually on a dedicated narrative)	✓ Less flexible solution as it is not based on an economic scenario generator (such a tool could be useful for other EIOPA purposes)	

Question to stakeholders : do you have some views regarding the best options ?

Methodological options to produce PHRSS

Step 2: Scenarios adjustments (1/4)

1 **Generation of raw scenarios**
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2 **Scenarios adjustments**
Correction of raw scenarios to meet martingale and market consistency properties

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✓ TVOG criterion: TVOG < 5% SCR ?
✓ With TVOG = Stochastic (PHRSS) BE – Deterministic (single scenario) BE

Principle: retreatments of raw scenarios to meet martingale and market consistency properties

Treatments to apply to raw scenarios:

- **Adjustment A** - Computation of moment matching techniques to adjust risk factors simulations in order to ensure convergence towards martingale tests targets
  - **Remark**: these adjustments are computed step by step (deflators, ZC prices, equity and real estate) on risk factors
  - Moment matching technique allows to obtain following martingale tests targets:

$$E\big(D(t)\big) = P(0,t)$$
$$E(D(t)P(t,T)) = P(0,T)$$
$$E(D(t)S(t)) = S(0)$$
$$E(D(t)RE(t)) = RE(0)$$

Example on equity risk

- Equity test target: $E(D(t)S(t)) = S(0)$
- Note $S^{adj}(t)$ the adjusted index defined by the dynamics: $S^{adj}(t) = S^{adj}(t-1) \times \frac{S^{init}(t)}{S^{init}(t-1)} \times AdjFactor_t$
- Below the formula to estimate adjustment factor:

$$AdjFactor_t = \frac{S(0)}{E\left(D^{adj}(t) \times S^{adj}(t-1) \times \frac{S^{init}(t)}{S^{init}(t-1)}\right)}$$

Methodological options to produce PHRSS

Step 2: Scenarios adjustments (2/4)

1 Generation of raw scenarios	2 Scenarios adjustments	3 Deterministic TP correction
Generation of an initial set of scenarios (almost 10 scenarios) by using risk-neutral models or real-world scenarios tailored by experts	Correction of raw scenarios to meet martingale and market consistency properties	✓ TVOG criterion: TVOG < 5% SCR ? ✓ With TVOG = Stochastic (PHRSS) BE – Deterministic (single scenario) BE

Treatments to apply to raw scenarios:

- **Adjustment B** - Scenarios reweighting to ensure the replication of derivative instruments prices targets (assessed by using **spot market data** or **prudent assumptions** on underlying volatilities)
  - The objective is to determine weights (probabilities) related to each simulation and allowing to obtain weighted average of discounted cash-flows **closer to derivative instruments** prices targets compared to **uniform weights**.
  - These **approaches** are developed in the **academic literature** (see for ex. M. AVELLANEDA "Weighted Monte Carlo: A New Technique for Calibrating Asset-Pricing Models")
    - Note that such a methodology is applied on a **full set of risk-neutral scenarios** and relies on complex optimization process (as it is not possible to directly optimize on weights vector in such a case due to dimension of underlying optimization to solve)
  - As the number of scenarios is limited in PHRSS framework, a direct optimization on weights does not raise any issues and leads to more efficient results. Such an approach consists in solving numerically the following program:

$$(p_1^*, \dots, p_{10}^*) = \underset{(p_1,\dots,p_{10})}{\text{ArgMin}} \sum_{c \in C} \left(\sum_{s=1}^{10} p_s CF_s^c - MarketPrice_c \right)^2$$

Methodological options to produce PHRSS

Step 2: Scenarios adjustments (3/4)

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Candidate methodologies for raw scenarios adjustments

- **Focus point**: adjustments **A** and **B** may be performed in different orders (which requires potential target function adaptation in **B**)
- **Method 2.1**: computation of **B** followed by **A** to meet martingale and market consistency requirements
- **Method 2.2**: computation of (i) **A** (to get a first set of coherent scenarios before reweighting) then (ii) **B** followed by (iii) **A** (to meet martingale and market consistency requirements)
- **Method 2.3**: adaptation of target function in **B** to encompass martingale and market consistency properties.
  - Below an example of modified target function:

$$(p_1^*, \dots, p_{10}^*) = \underset{(p_1,\dots,p_{10})}{\text{ArgMin}} \left\{ w_1 \sum_{c \in C} \left(\sum_{s=1}^{10} p_s CF_s^c - MarketPrice_c \right)^2 + w_2 \sum_t \left(E\left(\widehat{D(t)}\right) - P(0,t) \right)^2 + w_3 \sum_t \left(E\left(\widehat{D(t)S}(t)\right) - S(0) \right)^2 + w_4 \sum_t \left(E\left(\widehat{D(t)RE}(t)\right) - RE(0) \right)^2 \right\}$$

Methodological options to produce PHRSS

Step 2: Scenarios adjustments (4/4)

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Below a summary of pros and cons per option:

Step 2: Scenarios adjustments	Pros	Cons
Method 2.1 – Computation B-A	✓ Martingale properties are met	✓ May distort market consistency features
Method 2.2 – Computation A-B-A	✓ Martingale properties are met	✓ May distort market consistency features (potentially moderate effect due to initialization with A compared to 2.1)
Method 2.3 – Adaptation of target function	✓ May allow to jointly manage martingale and market consistency targets	✓ May slightly distort martingale properties
Method 2.4 – Adaptation of target function followed by A	✓ Martingale properties are met	✓ May slightly distort market consistency features but as martingale constraints are integrated within the target function, a final application of A may induce limited impacts

Question to stakeholders : do you have some views regarding the best options ?

Methodological options to produce PHRSS

Options overview

Below a summary of options mentioned previously:

Out of scope of slides

Step 1 - Generation of raw scenarios	Step 2 - Scenarios adjustments	Step 3 - Deterministic TP correction

✓ Method 1.1 - Pure stochastic trajectories
✓ Method 1.2 - Percentiles level lines
✓ Method 1.3 - Ranked scenarios
✓ Method 1.4 - Ranked scenarios with conditional expectations
✓ Method 1.5 - Real-world scenarios defined “by hand” by experts
	✓ Method 2.1 – Computation B-A
	✓ Method 2.2 – Computation A-B-A
	✓ Method 2.3 – adaptation of target function
	✓ Method 2.4 – Adaptation of target function followed by A
		✓ TVOG criterion: TVOG < 5% SCR ?
		    ✓ With TVOG = Stochastic (PHRSS) BE – Deterministic (single scenario) BE
		✓ For LRPU which meet TVOG criterion: adjustment of deterministic BE with
		    ✓ a supplement equal to 5% of SCR ;
		    ✓ Or an ad-hoc(*) stochastic supplement using PHRSS).

Reminder: generation of raw scenarios in Step 1 may rely on initial sets of **RN** or **RW scenarios** (except for the method 1.5)

(*) Subject to discussion with NCAs

Content

- Introduction – overview of the project
- Methodological options to produce PHRSS
- **General features of scenarios – discussion points**

Discussion points

General features of scenarios

Objective: parameterization of PHRSS scenarios

- Expected number of scenarios: approximatively 10 scenarios
- Type of risk factors: risk free rates, equity, real estate,…
- Time step: monthly or yearly
- Length of time horizon: 60 Years

Question to stakeholders :

- **What kind of risk factors should be included?**
- **What time step should be used?**
- **What time horizon should be considered?**

Discussion points

General features of scenarios

Objective: parameterization of PHRSS scenarios

- Expected outcomes: ZC prices vs rates, index values vs return,…
- Regular publication by EIOPA: in form of tables or by providing a tool

Question to stakeholders :

- **What is the preferred expected outcome for the scenarios?**
- **Should EIOPA publish tables on a regular basis, or a tool?**

Discussion points

General features of scenarios

Objective: parameterization of PHRSS scenarios

- Calibration of scenarios

Question to stakeholders :

- **Should the calibration be market consistent or should it be based on historical volatilities?**
- **Could a real-world model based on Solvency II standard formula shocks (see appendix) be used?**

Appendix 1
Zoom on Real-World models based on Solvency 2 shocks (1/2)

Below some examples of real-world models calibrated on Solvency 2 Standard Formula shocks

- **Interest rates (IR) model**
  - Gaussian dynamics centered on forward rates (parallel shift):

$$\tilde{r}(t,m) = r^f(t,m) + \sigma_{IR} \sum_{k=1}^{t} \varepsilon_k^{IR}$$

    - With $r^f(t,m)$ the forward rate at period t related to maturity m
    - And $\varepsilon_k^{IR} \approx N(0,1)$

  - Model calibration
    - Estimation of IR volatility parameter σ_{IR} based on replication of 1% absolute shock (level of upward shock observed in last closing exercises):

$$q_{99.5\%}(\sigma_{IR}.\varepsilon^{IR}) = 1\%$$

      - Which leads to $\sigma_{IR} = \frac{1\%}{q_{99.5\%}(\varepsilon^{IR})} \approx 0.39\%$

Appendix 1

Zoom on Real-World models based on Solvency 2 shocks (2/2)

Below some examples of real-world models calibrated on Solvency 2 Standard Formula shocks

- **Equity (EQ) model**
  - Log-normal dynamics of EQ index:

$$S(t) = S(t-1) \times \frac{1}{P(t-1,1)} \times e^{-0.5\sigma_{EQ}{}^2 + \sigma_{EQ}\varepsilon_t^{EQ}}$$

    - With: $\varepsilon_t^{EQ} \approx N(0,1)$
  - Model calibration
    - Estimation of EQ volatility parameter σ_{EQ} by replicating the 39% Solvency 2 shock (IR impact neglected):

$$e^{-0.5\sigma_{EQ}{}^2 + \sigma_{EQ}.q_{0.5\%}(N(0,1))} = 1 - 39\%$$

    - Which leads to $\sigma_{EQ} \approx 19\%$
- **Real Estate (RE) model**
  - Same approach than EQ risk factor
  - Estimation of RE volatility parameter based on 25% Solvency 2 shock replication, which leads to $\sigma_{EQ} \approx 11\%$

THANK YOU!

European Insurance and
Occupational Pensions Authority