



Calculus 1 Solutions – 12 Jan 2022

1)

Answers:

(i) $4x^3$ (ii) $21x^2$ (iii) $-x^{-2}$ (iv) 1

2)

(i) $f'(x) = 10x + 7$

(ii) $g'(x) = 6x^2 + 8x + 1$

(iii) $\frac{dy}{dx} = 8$

3)

$y = 8x + 4$

this is the equation of a line

$\frac{dy}{dx} = 8$ this is the slope of that line

Prove it: $y = mx + c$

$m = 8$

4)

Product Rule - Example

$y = (4 + 3x^2)(6x + 4x^2)$

$\frac{du}{dx} = 6x$ $\frac{dv}{dx} = 6 + 8x$

Find $\frac{dy}{dx}$

(hint let $u = (4 + 3x^2)$

and $v = (6x + 4x^2)$)

$\frac{dy}{dx} = (4 + 3x^2)(6 + 8x) + (6x + 4x^2)(6x)$

$\frac{dy}{dx} = 24 + 32x + 18x^2 + 24x^3 + 36x^2$

$+ 24x^3$

$= 48x^3 + 54x^2 + 32x + 24$

5) Quotient rule: $y = \frac{(5+6x)}{2x^2}$ **Find $\frac{dy}{dx}$**

Quotient Rule - Example

$y = \frac{(5 + 6x)}{2x^2}$

$\frac{du}{dx} = 6$ $\frac{dv}{dx} = 4x$

Find $\frac{dy}{dx}$

(hint let $u = (5 + 6x)$

and $v = 2x^2$)

$\frac{dy}{dx} = \frac{(2x^2)(6) - (5 + 6x)(4x)}{(2x^2)^2}$



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$$\frac{dy}{dx} = \frac{12x^2 - 20x - 24x^2}{4x^4}$$

$$\frac{dy}{dx} = \frac{-12x^2 - 20x}{4x^4}$$

(divide above and below by $4x$)

$$\frac{dy}{dx} = \frac{-3x - 5}{x^3}$$

6) Chain rule: $y = (2 + 3x)^4$ Find $\frac{dy}{dx}$

Chain Rule - Example

$$y = (2 + 3x)^4$$

Find $\frac{dy}{dx}$

(hint let $v = (2 + 3x)$)

and $u = (v)^4$

$$\frac{du}{dv} = 4v^3 \quad \frac{dv}{dx} = 3$$

$$\begin{aligned} f'(x) &= (4v^3)(3) \\ &= (4(2 + 3x)^3)(3) \\ &= (4(2 + 3x)^3)(3) \\ &= 12(2 + 3x)^3 \end{aligned}$$

7) Find the following limits:

$$a) \lim_{x \rightarrow 1} \left(\frac{x^2 + x - 2}{x - 1} \right) = \lim_{x \rightarrow 1} \left(\frac{(x-1)(x+2)}{x-1} \right) = \lim_{x \rightarrow 1} (x + 2) = 1 + 2 = 3$$

$$b) \lim_{n \rightarrow \infty} \left(\frac{2n^2 - 3n + 2}{6n^2 + 5n - 6} \right) = \lim_{n \rightarrow \infty} \left(\frac{2 - \frac{3}{n} + \frac{2}{n^2}}{6 + \frac{5}{n} - \frac{6}{n^2}} \right) = \frac{2}{6} = \frac{1}{3}$$

8) Find the derivative of $f(x) = 5 - 2x$ by first principles.

First find $f(x + h)$

$$\begin{aligned} f(x + h) &= 5 - 2(x + h) \\ &= 5 - 2x - 2h \end{aligned}$$

$$\begin{aligned} \frac{f(x+h) - f(x)}{h} &= \frac{5 - 2x - 2h - 5 + 2x}{h} \\ &= -2 \end{aligned}$$

$$\frac{f(x + h) - f(x)}{h} = \frac{5 - 2x - 2h - 5 + 2x}{h}$$

9) Fill in the table:



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$f(x)$	$f'(x)$
$\sin^{-1}\left(\frac{x}{a}\right)$	$\frac{1}{\sqrt{a^2 - x^2}}$
$\cos^{-1}\left(\frac{x}{a}\right)$	$-\frac{1}{\sqrt{a^2 - x^2}}$
$\tan^{-1}\left(\frac{x}{a}\right)$	$\frac{a}{a^2 + x^2}$

10)

- (a) Differentiate the function $2x^2 - 3x - 6$ with respect to x from first principles.

$$f(x) = 2x^2 - 3x - 6$$

$$f(x+h) = 2(x+h)^2 - 3(x+h) - 6 = 2x^2 + 4xh + 2h^2 - 3x - 3h - 6$$

$$f(x+h) - f(x) = 4xh + 2h^2 - 3h$$

$$\text{Limit}_{h \rightarrow 0} \left(\frac{f(x+h) - f(x)}{h} \right) = \text{Limit}_{h \rightarrow 0} \left(\frac{4xh + 2h^2 - 3h}{h} \right) = 4x - 3$$



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- (b) Let $f(x) = \frac{2x}{x+2}$, $x \neq -2$, $x \in \mathbb{R}$. Find the co-ordinates of the points at which the slope of the tangent to the curve $y = f(x)$ is $\frac{1}{4}$.

$$f(x) = \frac{2x}{x+2}$$

$$\text{Let } u(x) = 2x \Rightarrow u'(x) = 2 \text{ and } v(x) = x+2 \Rightarrow v'(x) = 1$$

$$f'(x) = \frac{(x+2)(2) - 2x(1)}{(x+2)^2} = \frac{4}{(x+2)^2}$$

$$f'(x) = \frac{1}{4} \Rightarrow \frac{4}{(x+2)^2} = \frac{1}{4}$$

$$\Rightarrow 16 = (x+2)^2$$

$$\Rightarrow x+2 = 4 \text{ or } x+2 = -4$$

$$\Rightarrow x = 2 \text{ or } x = -6$$

or

$$x^2 + 4x - 12 = 0$$

$$(x-2)(x+6) = 0$$

$$\Rightarrow x-2 = 0 \text{ or } x+6 = -0$$

$$\Rightarrow x = 2 \text{ or } x = -6$$

$$f(-6) = \frac{-12}{-6+2} = 3 \text{ and } f(2) = \frac{4}{2+2} = 1$$

Points $(-6, 3)$ and $(2, 1)$



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11)

Q6	Model Solution – 25 Marks	Marking Notes
(a)	$f(x+h) - f(x) = (2x+2h+4)^2 - (2x+4)^2$ $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} =$ $\lim_{h \rightarrow 0} \frac{(2x+2h+4)^2 - (2x+4)^2}{h}$ $= \lim_{h \rightarrow 0} \left(\frac{[(4x^2 + 8hx + 4h^2 + 16x + 16h + 16)] - (4x^2 + 16x + 16)}{h} \right)$ $= \lim_{h \rightarrow 0} \frac{8hx + 4h^2 + 16h}{h}$ $= 8x + 16$ <p style="text-align: center;">or</p> $f(x) = (2x+4)^2 = 4x^2 + 16x + 16$ $f(x+h) = 4(x+h)^2 + 16(x+h) + 16$ $= 4x^2 + 8hx + 4h^2 + 16x + 16h + 16$ $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $\lim_{h \rightarrow 0} \frac{8hx + 4h^2 + 16h}{h}$ $= 8x + 16$	<p>Scale 10D (0, 2, 5, 8, 10)</p> <p><i>Low Partial Credit</i></p> <ul style="list-style-type: none"> any $f(x+h)$ <p><i>Mid Partial Credit</i></p> <ul style="list-style-type: none"> limit of $\frac{f(x+h) - f(x)}{h}$ <p><i>High Partial Credit</i></p> <ul style="list-style-type: none"> limit of $\frac{(2x+2h+4)^2 - (2x+4)^2}{h}$ <p>Notes:</p> <ul style="list-style-type: none"> omission of limit sign penalised once only answer not from 1st Principles merits 0 marks

(b) (i)+ (ii)	$y = x \cdot \sin \frac{1}{x}$ $\frac{dy}{dx} = \sin \frac{1}{x} + x \left(\cos \frac{1}{x} \right) \left(-\frac{1}{x^2} \right)$ $\frac{dy}{dx} = \sin \frac{1}{x} - \frac{1}{x} \cos \frac{1}{x}$ $\frac{dy}{dx} = \sin \frac{\pi}{4} - \frac{\pi}{4} \cos \frac{\pi}{4}$ $= 0.15$	<p>Scale 15D (0, 4, 7, 11, 15)</p> <p><i>Low Partial Credit</i></p> <ul style="list-style-type: none"> any correct differentiation <p><i>Mid Partial Credit</i></p> <ul style="list-style-type: none"> product rule applied <p><i>High Partial Credit</i></p> <ul style="list-style-type: none"> correct differentiation <p>Note: one penalty for calculator in wrong mode</p>
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12)

Q3	Model Solution – 25 Marks	Marking Notes
(a)	$f(x+h) = \frac{1}{3}(x+h)^2 - (x+h) + 3$ $f(x) = \frac{1}{3}x^2 - x + 3$ $f(x+h) - f(x) = \frac{2xh}{3} + \frac{h^2}{3} - h$ $\frac{f(x+h) - f(x)}{h} = \frac{2x}{3} + \frac{h}{3} - 1$ $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \frac{2x}{3} - 1$	<p>Scale 20D (0, 5, 14, 17, 20)</p> <p><i>Low Partial Credit</i></p> <ul style="list-style-type: none"> any $f(x+h)$ <p><i>Mid Partial Credit</i></p> <ul style="list-style-type: none"> $f(x+h) - f(x)$ with some correct work <p><i>High Partial Credit</i></p> <ul style="list-style-type: none"> $\frac{\frac{1}{3}(x+h)^2 - (x+h) + 3 - (\frac{1}{3}x^2 - x + 3)}{h}$ simplified <p>Notes:</p> <ul style="list-style-type: none"> omission of limit sign penalised once only answer not from 1st Principles merits 0 marks
(b)	$\frac{d(fg(x))}{dx} =$ $\frac{1}{(3(x+5)^2 + 2)} (6(x+5))$ $\frac{d(fg(\frac{1}{4}))}{dx} = \frac{6(\frac{21}{4})}{3(\frac{21}{4})^2 + 2} = \frac{504}{1355}$ $= 0.372$ <p style="text-align: center;">OR</p> $f(x) = \ln(3x^2 + 2)$ $g(x) = (x + 5)$ $f[g(x)] = \ln[3(x+5)^2 + 2]$ $= \ln(3x^2 + 30x + 77)$ $f'(x) = \frac{6x + 30}{3x^2 + 30x + 77}$ $x = \frac{1}{4}: f'(x) = \frac{31.5}{84.6875} = 0.3719$ $= 0.372$	<p>Scale 5C (0, 3, 4, 5)</p> <p><i>Low Partial Credit:</i></p> <ul style="list-style-type: none"> Any correct differentiation $fg(x)$ formulated <p><i>High Partial Credit:</i></p> <ul style="list-style-type: none"> $\frac{d(fg(x))}{dx}$ found <p>Note:</p> <p>Work with $f(x) \times g(x)$ merits low partial credit at most</p>