

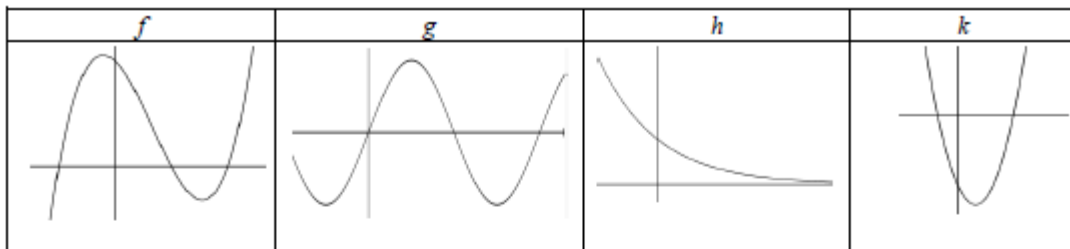


Calculus 2 Tutorial Questions 2021/2022

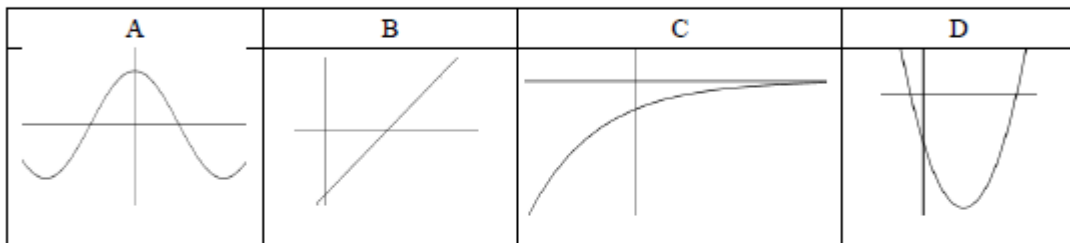
Question 1

[2013 Sample Paper 1, Q5] (25 marks)

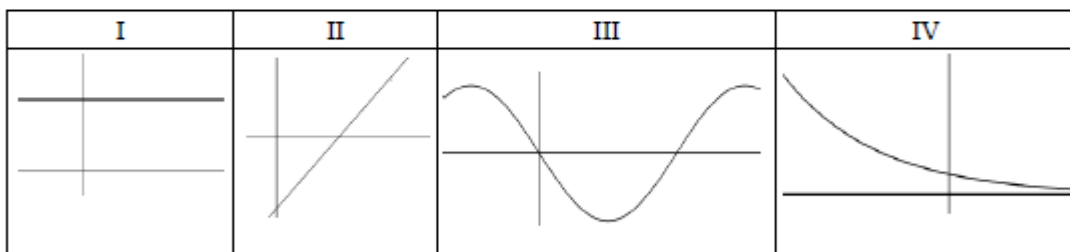
Each diagram below shows part of the graph of a function. Each of these functions is either quadratic or cubic or trigonometric or exponential (not necessarily in that order).



Each diagram below shows part of the graph of the first derivative of one of the above functions (not necessarily in the same order).



Each diagram below shows part of the graph of the second derivative of one of the original functions (not necessarily in the same order).



(a) Complete the table below by matching the function to its first derivative and its second derivative.

Type of function	Function	First derivative	Second derivative
Quadratic			
Cubic			
Trigonometric			
Exponential			

(b) For one row in the table, explain your choice of first derivative and second derivative.

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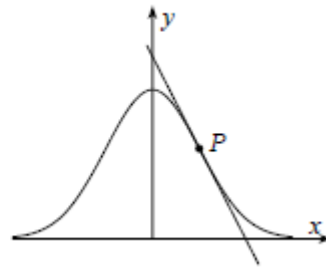
Question 2

[2012 Paper 1, Q6] (25 marks)

(a) Let $f(x) = e^{-\frac{1}{2}x^2}$.

Show that the second derivative of $f(x)$ with respect to x is $f''(x) = (x^2 - 1)e^{-\frac{1}{2}x^2}$.

- (b) The point P in the first quadrant is a point of inflection of the curve $y = e^{-\frac{1}{2}x^2}$.
Show that the tangent at P crosses the x -axis at $(2, 0)$.



Question 3

[2015 Paper 1, Q3] (25 marks)

Let $f(x) = -x^2 + 12x - 27$, $x \in \mathbb{R}$.

- (a) (i) Complete Table 1 below.

Table 1							
x	3	4	5	6	7	8	9
$f(x)$	0	5			8		

- (ii) Use Table 1 and the trapezoidal rule to find the approximate area of the region bounded by the graph of f and the x -axis.
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(b) (i) Find $\int_3^9 f(x) dx$.

- (ii) Use your answers above to find the percentage error in your approximation of the area, correct to one decimal place.
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Question 4

[2014 Paper 1, Q5] (25 marks)

- (a) Find $\int 5\cos 3x \, dx$.
- (b) The slope of the tangent to a curve $y = f(x)$ at each point (x, y) is $2x - 2$.
The curve cuts the x -axis at $(-2, 0)$.
- (i) Find the equation of $f(x)$.
- (ii) Find the average value of f over the interval $0 \leq x \leq 3, x \in \mathbb{R}$.
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Question 5

[2015 Paper 1, Q5] (25 marks)

- (a) Solve the equation $x = \sqrt{x+6}, x \in \mathbb{R}$.
- (b) Differentiate $x - \sqrt{x+6}$ with respect to x .
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- (c) Find the co-ordinates of the turning point of the function $y = x - \sqrt{x+6}, x \geq -6$.
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Question 6

[2016 Paper 1, Q7] (40 marks)

- (a) (i) Air is pumped into a spherical exercise ball at the rate of 250 cm^3 per second. Find the rate at which the radius is increasing when the radius of the ball is 20 cm. Give your answer in terms of π .
- (ii) Find the rate at which the surface area of the ball is increasing when the radius of the ball is 20 cm.
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- (b) The inflated ball is kicked into the air from a point O on the ground. Taking O as the origin, $(x, f(x))$ approximately describes the path followed by the ball in the air, where

$$f(x) = -x^2 + 10x$$

and both x and $f(x)$ are measured in metres.

- (i) Find the values of x when the ball is on the ground.
- (ii) Find the average height of the ball above the ground, during the interval from when it is kicked until it hits the ground again.





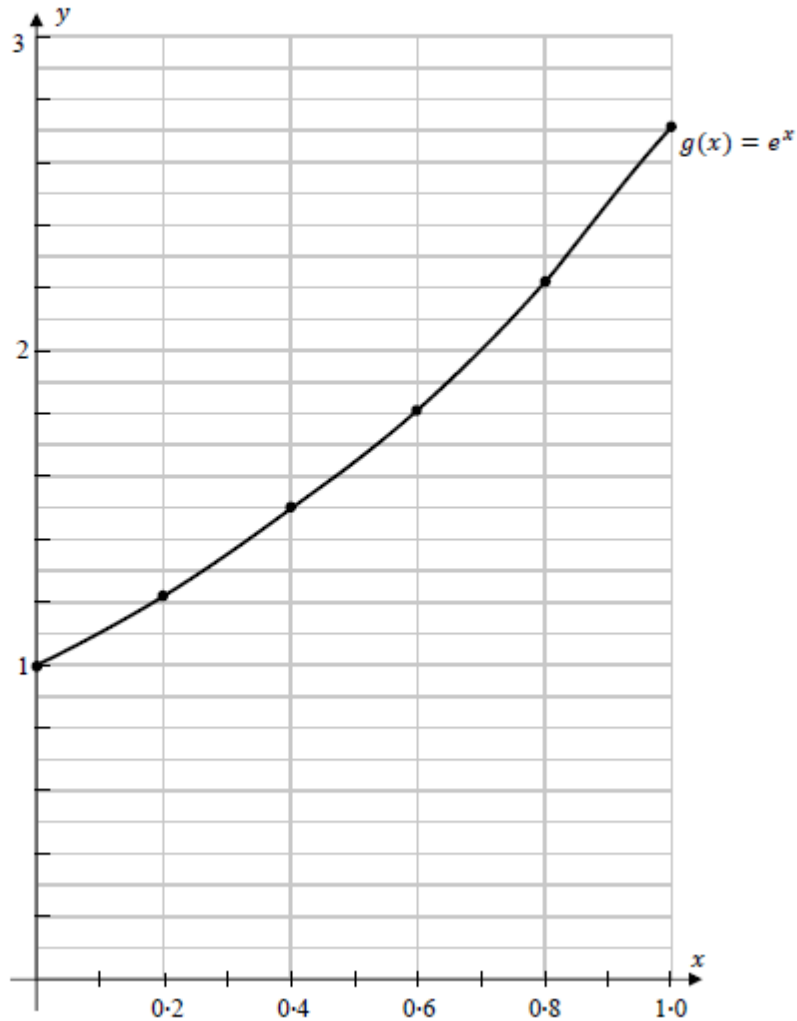
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Question 7

[2017 Paper 1, Q6] (25 marks)

The graph of the function $g(x) = e^x$, $x \in \mathbb{R}$, $0 \leq x \leq 1$, is shown on the diagram below.

(a) On the same diagram, draw the graph of $h(x) = e^{-x}$, $x \in \mathbb{R}$, in the domain $0 \leq x \leq 1$.



(b) Find the area enclosed by $g(x) = e^x$, $h(x) = e^{-x}$, and the line $x = 0.75$.
Give your answer correct to 4 decimal places.

Solutions: <https://web.actuaries.ie/students/maths-tutorials-higher-level-leaving-certificate-20212022> or google 'actuaries maths tutorials'