



Trigonometry 2 – Solutions

Trigonometric formulae

Question 1.

We have a mixture of functions of A and $7A$ on the left side of this equation, which makes it difficult to start solving this equation. We will have to use the formulae for $\cos(A \pm B)$ and $\sin(A \pm B)$ to simplify it. If we let $x = 4A$ and $y = 3A$ then we can express $7A$ and A in terms of these:

$$7A = x + y$$

$$A = x - y$$

$$\begin{aligned} \Rightarrow & (\cos 7A + \cos A) / (\sin 7A - \sin A) \\ &= (\cos(x+y) + \cos(x-y)) / (\sin(x+y) - \sin(x-y)) \\ &= ([\cos x \cos y - \sin x \sin y] + [\cos x \cos y + \sin x \sin y]) \\ &\quad / ([\sin x \cos y + \cos x \sin y] - [\sin x \cos y - \cos x \sin y]) \\ &= (2\cos x \cos y) / (2 \cos x \sin y) \\ &= (\cos y) / (\sin y) \\ &= 1 / \tan y \\ &= \cot y \\ &= \cot 3A \end{aligned}$$

Radians

Question 2.

Each runner runs a part of a circle (an arc) with angle θ . Let r be the radius (in metres) of the circle Kate runs on. Then the length of the arc that Kate runs is $r\theta$ metres.

Since the lanes are both 1.2 m wide, the midpoints of the lanes must be 1.2 m apart, so the radius of the circle Helen runs must be $(r + 1.2)$ m. Then the length of the arc that Helen runs is $(r+1.2)\theta$ metres.

\Rightarrow Helen runs 1.2θ metres further than Kate.

$$\Rightarrow 1.2\theta = 3$$

$$\Rightarrow \theta = 2.5 \text{ rad}$$



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Solving equations

Question 3.

Let $x = \cos \theta$

We can use the trigonometric formulae to express $\cos 2\theta$ in terms of $\cos \theta$ and $\sin \theta$, and then eliminate $\sin \theta$ so we express it purely in terms of $\cos \theta$. (Recall that $\cos^2 \theta$ just means the square of $\cos \theta$.)

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta = \cos^2 \theta - (1 - \cos^2 \theta) = 2\cos^2 \theta - 1 = 2x^2 - 1$$

$$\Rightarrow 2x^2 - 1 = 1/9$$

$$\Rightarrow 2x^2 = 10/9$$

$$\Rightarrow x^2 = 5/9$$

$$\Rightarrow x = \pm\sqrt{5}/3$$

ANSWER: $\cos \theta = \pm\sqrt{5}/3$



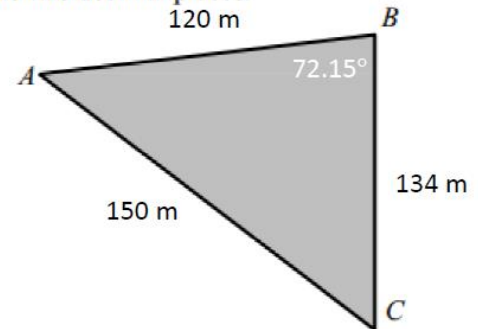
Trigonometry 2 – Solutions

Sine rule, cosine rule, area of a triangle

Question 4.

- (a) (i) Find $|\angle CBA|$. Give your answer, in degrees, correct to two decimal places.

$$\begin{aligned}\cos B &= \frac{a^2 + c^2 - b^2}{2ac} \\ &= \frac{120^2 + 134^2 - 150^2}{2(120)(134)} \\ &= \frac{9856}{32160} \\ &= 0.306468 \\ \Rightarrow B &= 72.15^\circ\end{aligned}$$



- (ii) Find the area of the triangle ACB correct to the nearest whole number.

$$\begin{aligned}\text{Area } \triangle ABC &= \frac{1}{2} ac \sin B = \frac{1}{2} (120)(134) \sin 72.15 \\ &= 7652.97 \\ &\approx 7653 \text{ m}^2\end{aligned}$$



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Question 5.

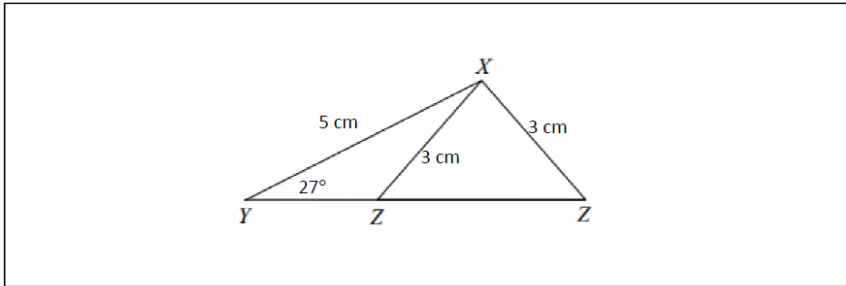
- (i) Find the two possible values of $|\angle XZY|$. Give your answers correct to the nearest degree.

$$\frac{3}{\sin 27^\circ} = \frac{5}{\sin \angle Z} \Rightarrow \sin \angle Z = \frac{5 \sin 27^\circ}{3} = 0.756$$

$$\Rightarrow |\angle Z| = 49^\circ \quad \text{or} \quad |\angle Z| = 131^\circ$$

- $\text{Sin}^{-1}(0.756) = 49.11^\circ$
- Sin is positive in the 1st and 2nd quadrants
- $180^\circ - 49^\circ = 131^\circ$

- (ii) Draw a sketch of the triangle XYZ , showing the two possible positions of the point Z .



In the case that $|\angle XZY| < 90^\circ$, write down $|\angle ZXY|$, and hence find the area of the triangle XYZ , correct to the nearest integer.

$$|\angle ZXY| = 180^\circ - (27^\circ + 49^\circ) = 104^\circ$$

$$\Delta = \frac{1}{2} ab \sin C = \frac{1}{2} (5)(3) \sin 104^\circ = 7.27 = 7 \text{ cm}^2$$

- Use 49° as $49^\circ < 90^\circ$
- 27° is the angle at Y
- All angles must sum to 180° so the remaining angle must equal 104°



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Longer questions

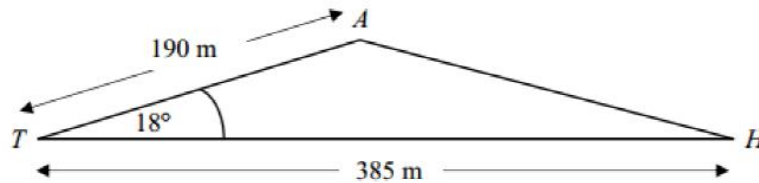
Question 6.

(a)

$$\begin{aligned} \sin \frac{1}{2}\alpha &= \frac{15}{150} = 0.1 \\ \Rightarrow \frac{1}{2}\alpha &= 5.739^\circ \\ \Rightarrow \alpha &= 11.478^\circ \\ \alpha &= 11.5^\circ \end{aligned}$$

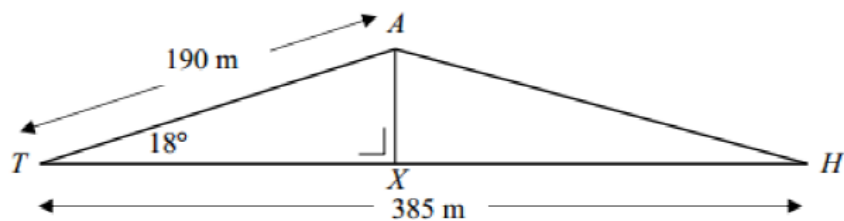
$$\sin^{-1} 0.1 = 5.739^\circ$$

(b)



$$\begin{aligned} |AH|^2 &= 190^2 + 385^2 - 2(190)(385)\cos 18^\circ \\ &= 36100 + 148225 - 139139 \cdot 5683 \\ &= 45185.4317 \\ |AH| &= 212.57 = 213 \end{aligned}$$

Part (b) – alternative solution by splitting triangle into two right triangles:



Draw AX perpendicular to TH

triangle ATX: $\sin 18^\circ = \frac{|AX|}{190} \Rightarrow |AX| = 58.71$

$$\cos 18^\circ = \frac{|TX|}{190} \Rightarrow |TX| = 180.7$$

$$\Rightarrow |XH| = 204.3$$

$$\Rightarrow |AH|^2 = (58.71)^2 + (204.3)^2$$

$$\Rightarrow |AH| = 212.566 = 213$$



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(c)

(i)

The ball is at K at time $t=0$ so the height of K is $h(0) = 8$.

Answer: 8 m

(ii)

The ball reaches the point B when its height is zero. This occurs at a time t that satisfies $h(t)=0$.

$$-6t^2 + 22t + 8 = 0$$

Solve this using the quadratic formula. The solutions are 4 and $-1/3$. We take the positive solution $t=4$.

The horizontal speed is 38 m/s.

The travel time is 4 s.

\Rightarrow The horizontal distance is $38 \times 4 = 152$ m.

Let α be the angle of elevation. Then $\tan(\alpha) = 8 / 152$.

$$\tan^{-1}(8 / 152) = 3.01^\circ$$

Answer: 3°

(d)

(i)

$$\tan(\theta) = \frac{1}{2} \Rightarrow h/d = \frac{1}{2}$$

$$\Rightarrow d = 2h$$

From the diagram we can see that:

$$|CD| = 25 - h$$

(ii)

We have a right triangle GDC with sides of length $2h$, $(25-h)$ and 25 (the hypotenuse). By Pythagoras:

$$25^2 = (2h)^2 + (25 - h)^2$$

$$= 4h^2 + 25^2 + h^2 - 50h$$

$$\Rightarrow 5h^2 = 50h$$

$$\Rightarrow h = 10$$



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Question 7.

<p>(a) (i)</p>	$ EC ^2 = 3^2 + 2 \cdot 5^2 = 15 \cdot 25$ $ EC = \sqrt{15 \cdot 25}$ $ EC = 3 \cdot 905$ $\Rightarrow AC = 1 \cdot 9525$ $= 1 \cdot 95$
<p>(a) (ii)</p>	$\tan 50^\circ = \frac{ AB }{1 \cdot 95}$ $ AB = 1 \cdot 95(1 \cdot 19175) = 2 \cdot 23239$ $ AB = 2 \cdot 3$



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(a)

(iii)

$$\begin{aligned} |BC|^2 &= 1.95^2 + 2.3^2 \\ |BC| &= 3.015377 \\ |BC| &= 3 \end{aligned}$$

Also: $\sin 40^\circ = \frac{1.95}{|BC|}$ or $\cos 40^\circ = \frac{2.3}{|BC|}$ or

$\cos 50^\circ = \frac{1.95}{|BC|}$ or $\sin 50^\circ = \frac{2.3}{|BC|}$

(a)

(iv)

$$3^2 = 3^2 + 2.5^2 - 2(3)(2.5) \cos \alpha$$

$$15 \cos \alpha = 6.25$$

$$\alpha = 65^\circ$$

or

$$\cos \alpha = \frac{1.25}{3}$$

$$\alpha = 65^\circ$$



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(a)

(v)

$A = 2 \times$ isosceles triangle $+ 2 \times$ equilateral triangle

$$= 2 \times \left[\frac{1}{2} (2.5)(3) \sin 65^\circ \right] +$$

$$2 \times \left[\frac{1}{2} (3)(3) \sin 60^\circ \right]$$

$$= 14.59$$

$$A = 15$$

(b)

$$\tan 60^\circ = \frac{3}{|CA|}$$

$$\Rightarrow |CA| = \sqrt{3}$$

$$|CE| = 2\sqrt{3}$$

$$x^2 + x^2 = (2\sqrt{3})^2$$

$$x = \sqrt{6}$$