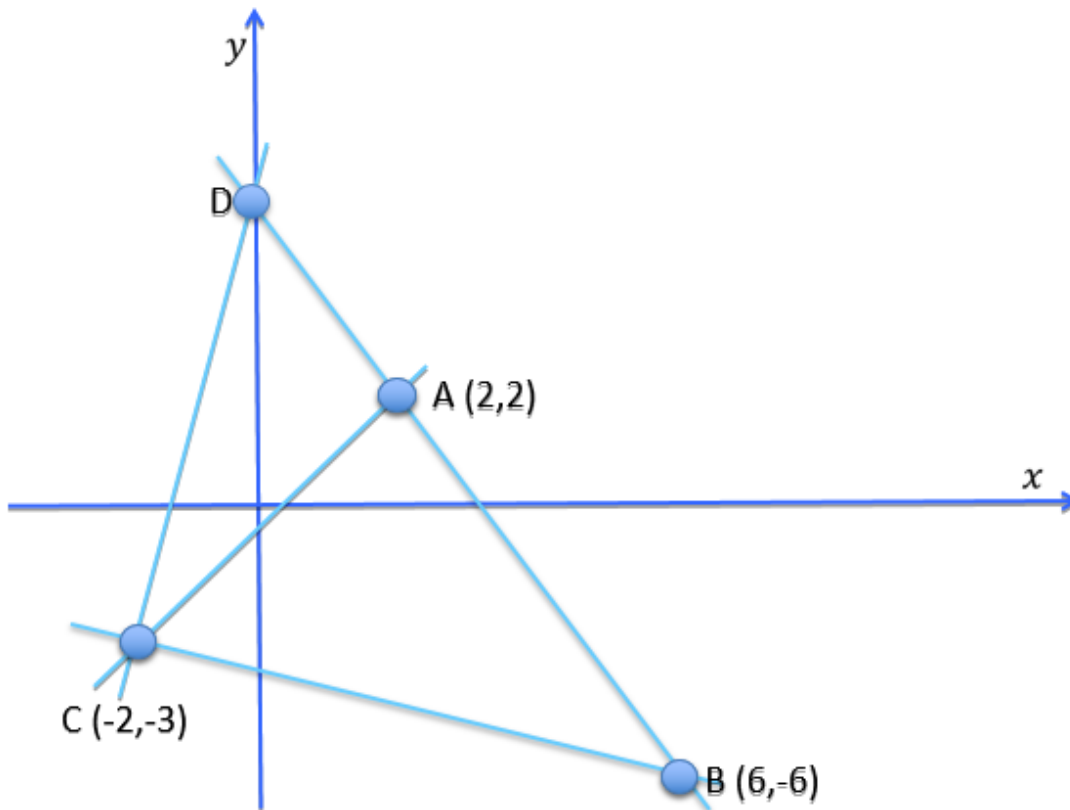


Geometry 2 Tutorial – Solutions

Q1.



a) Find the slope using slope formula

$$\Rightarrow m = \frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{(-6 - 2)}{(6 - 2)} = \frac{-8}{4} = -2$$

$$\Rightarrow \text{Slope} = -2, A = (2, 2)$$

$$\Rightarrow \text{Equation of AB: } y - 2 = -2 * (x - 2)$$

$$\Rightarrow y - 2 = -2x + 4$$

$$\Rightarrow 2x + y = 6$$

b) Line intersects the y-axis means that $x=0$ at this point

$$\Rightarrow \text{Substitute } x=0 \text{ into Equation of AB: } 2x + y = 6$$

$$\Rightarrow y = 6$$

$$\Rightarrow D = (0, 6)$$



c) Formula in Tables Page 19

$$C = (-2, -3) \text{ and } AB = 2x + y = 6 \text{ or } 2x + y - 6 = 0$$

$$\Rightarrow \text{Distance} = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}} = \frac{|2(-2) + 1(-3) - 6|}{\sqrt{2^2 + 1^2}} = \frac{|-13|}{\sqrt{5}} = \frac{13}{\sqrt{5}}$$

d) Area of Triangle is $\frac{1}{2} \times \text{base} \times \text{perpendicular height}$

\Rightarrow Perpendicular height is $\frac{13}{\sqrt{5}}$ from last question

\Rightarrow Distance between A(2,2) and D(0,6) is the base

$$\Rightarrow \sqrt{(0 - 2)^2 + (6 - 2)^2}$$

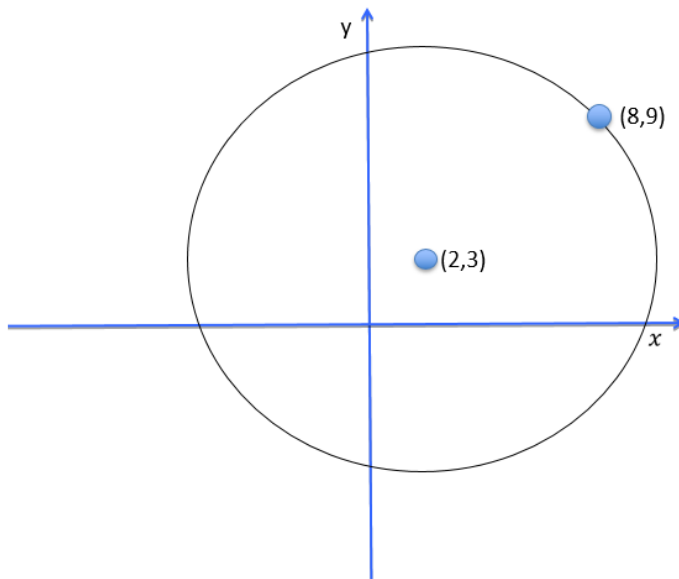
$$\Rightarrow \sqrt{4 + 16} = \sqrt{20}$$

$$\Rightarrow \text{Area} = \frac{1}{2} * \sqrt{20} * \frac{13}{\sqrt{5}} = 13 \text{ units squared}$$

Note: To use formula for area of triangle on page 18 it is necessary to have one apex at the origin.

Q2.

a)



b) Distance between (8,9) and (2,3) is radius

$$\Rightarrow \sqrt{(2-8)^2 + (3-9)^2}$$

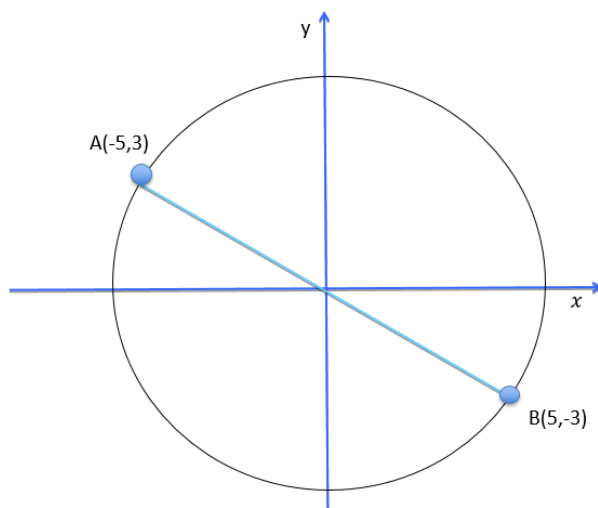
$$\Rightarrow \sqrt{(-6)^2 + (-6)^2} = \sqrt{36 + 36} = \sqrt{72}$$

c) Equation of circle with centre (h,k) and radius r is: $(x-h)^2 + (y-k)^2 = r^2$

$$\Rightarrow (x-2)^2 + (y-3)^2 = \sqrt{72}^2 = 72$$

Q3.

a)



b) Midpoint of A(-5,3) and B(5,-3) is the centre of the circle

$$\Rightarrow \text{Midpoint is } (0,0)$$

c) Distance between (-5,3) and (0,0) is radius

$$\Rightarrow \sqrt{(-5-0)^2 + (3-0)^2}$$

$$\Rightarrow \sqrt{5^2 + 3^2}$$

$$\Rightarrow \sqrt{34}$$

d) $x^2 + y^2 = r^2$

$$\Rightarrow x^2 + y^2 = (\sqrt{34})^2$$

$$\Rightarrow x^2 + y^2 = 34$$

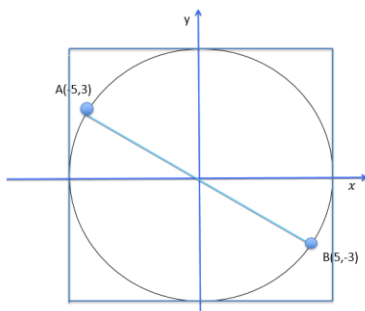
e) $Area = \pi r^2$

$$\Rightarrow \pi \sqrt{34}^2$$

$$\Rightarrow 34 \pi$$

$$\Rightarrow 106.81 \text{ units squared}$$

f)



Side of square is equal to the diameter, area of square is diameter²

$$\Rightarrow \text{Diameter} = 2r = 2\sqrt{34}$$

$$\Rightarrow \text{Area of square} = (2\sqrt{34})^2$$

$$\Rightarrow 136 \text{ units squared}$$

Q4.

a) *Solution circle c1*

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

$$\Rightarrow x^2 + y^2 - 6x - 10y + 29 = 0 \quad (\text{Formulae p. 19})$$

$$\Rightarrow 2g = -6$$

$$\Rightarrow -g = 3$$

$$\Rightarrow 2f = -10$$

$$\Rightarrow -f = 5$$

$$\Rightarrow \text{centre } (3, 5)$$

$$\Rightarrow \text{Radius formula (p. 19)}$$

$$\Rightarrow \sqrt{(g^2 + f^2 - c)}, c = 29$$

$$\Rightarrow \sqrt{3^2 + 5^2 - 29}$$

$$\Rightarrow \sqrt{9 + 25 - 29}$$

$$\Rightarrow \sqrt{5}$$

Solution circle c2

$$x^2 + y^2 - 2x - 2y - 43 = 0$$

$$\Rightarrow 2g = -2$$

$$\Rightarrow -g = 1$$

$$\Rightarrow 2f = -2$$

$$\Rightarrow -f = 1$$

$$\Rightarrow \text{centre } (1,1)$$

$$\Rightarrow \text{Radius formula}$$

$$\Rightarrow \sqrt{1^2 + 1^2 + 43}$$

$$\Rightarrow \sqrt{45}$$

$$\Rightarrow \sqrt{9 * 5}$$

$$\Rightarrow 3\sqrt{5}$$



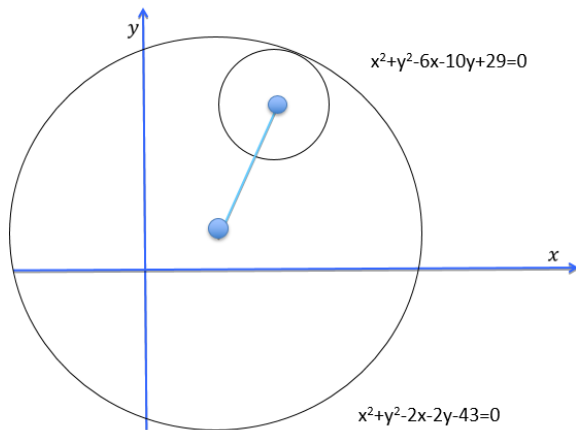
b) Distance between centres:

$$\Rightarrow \sqrt{(3-1)^2 + (5-1)^2}$$

$$\Rightarrow \sqrt{20}$$

$$\Rightarrow 2\sqrt{5}$$

\Rightarrow The distance between the centres is the difference of the radii \Rightarrow circles touch (internally).



c) Substitute $(4, 7)$ into c_1 and c_2

$$\Rightarrow 4^2 + 7^2 - 6(4) - 10(7) + 29 = 0$$

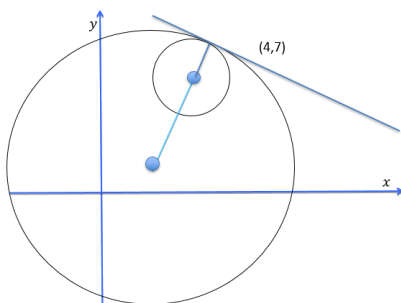
$$\Rightarrow (4, 7) \text{ is point on } c_1$$

$$\Rightarrow 4^2 + 7^2 - 2(4) - 2(7) - 43 = 0$$

$$\Rightarrow (4, 7) \text{ is point on } c_2$$

\Rightarrow Circles touch internally at $(4, 7)$

d)





Slope from $(3, 5)$ to $(4, 7)$ is: $\frac{7-5}{4-3} = 2$

\Rightarrow Slope of tangent = $-\frac{1}{2}$ (as perpendicular)

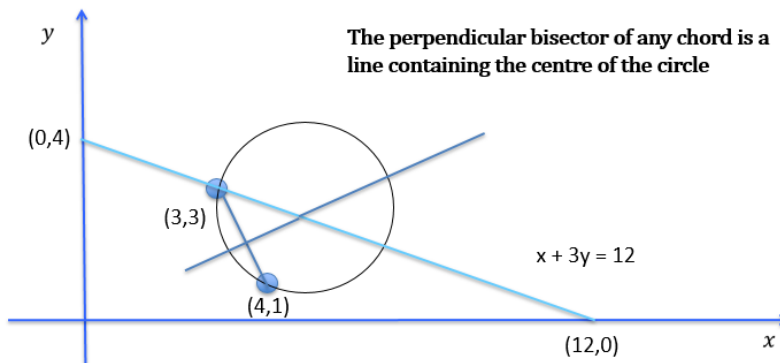
\Rightarrow Equation of tangent with slope $-\frac{1}{2}$ and point $(4,7)$

$\Rightarrow y - 7 = -\frac{1}{2}(x - 4)$

$\Rightarrow 2y - 14 = -x + 4$

$\Rightarrow x + 2y - 18 = 0$

Q5.



The perpendicular bisector of any chord is a line containing the centre of the circle

\Rightarrow End points of chord: $(3,3)$ and $(4,1)$

\Rightarrow Midpoint of Chord: $(3.5, 2)$

\Rightarrow slope of chord: $(1-3)/(4-3) = -2$

\Rightarrow slope of perpendicular of chord: $\frac{1}{2}$

Eqn of perpendicular of chord: slope = $\frac{1}{2}$, point $(3.5, 2)$

$\Rightarrow y - 2 = \frac{1}{2}(x - 3.5)$

$\Rightarrow 2y - 4 = x - 3.5$

$\Rightarrow x - 2y = -0.5$

\Rightarrow $x + 3y = 12$ point of intersection is centre of circle



$$\Rightarrow -5y = -12.5 \text{ subtracting}$$

$$\Rightarrow y = 2.5; x = 4.5$$

$$\Rightarrow \text{Centre is } (4.5, 2.5)$$

Now, need to find the radius

$$\Rightarrow \text{Distance between } (4.5, 2.5) \text{ and } (3, 3)$$

$$\Rightarrow r = \sqrt{(3 - 4.5)^2 + (3 - 2.5)^2}$$

$$\Rightarrow r = \sqrt{2.5}$$

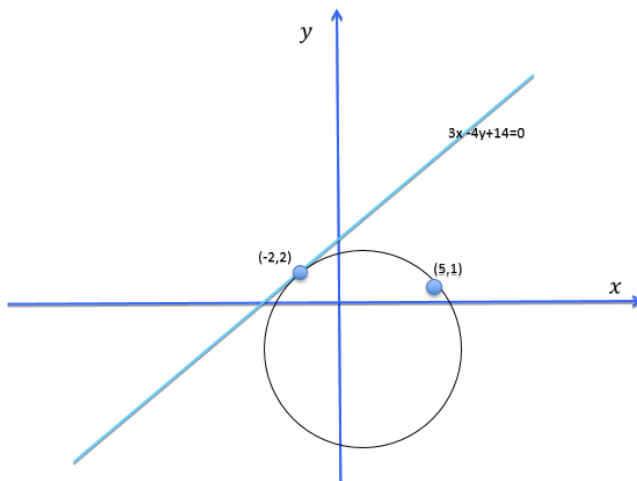
Equation of the circle:

$$\Rightarrow (x - 4.5)^2 + (y - 2.5)^2 = 2.5$$

$$\Rightarrow x^2 + y^2 - 9x - 5y + 24 = 0$$

Q6.

a)



b) Need to find the centre of the circle to find the equation of the circle

Know: The radius is perpendicular to the tangent at the point of intersection

Know: The perpendicular bisector of any chord is a line containing the centre of the circle

⇒ The point of intersection of these two lines is the centre of the circle

Tangent: $3x - 4y + 14 = 0$

⇒ Slope of tangent = $\frac{3}{4}$

⇒ slope of perpendicular = $-\frac{4}{3}$

Equation of perpendicular: slope $-\frac{4}{3}$, point $(-2, 2)$

⇒ $y - 2 = -\left(\frac{4}{3}\right)(x - (-2))$

⇒ $3y - 6 = -4x - 8$

⇒ $4x + 3y = -2$

To find perpendicular bisector of chord:

⇒ Midpoint $(-2, 2)$ and $(5, 1)$: $(1.5, 1.5)$

⇒ Slope of $(-2, 2)$ and $(5, 1)$: $(1-2)/(5-(-2))$

⇒ Slope: $-\frac{1}{7}$

⇒ Slope of perpendicular = 7

Equation of perpendicular: slope 7, point $(1.5, 1.5)$

⇒ $y - 1.5 = 7(x - 1.5)$

⇒ $7x - y = 9$

Simultaneous equations

⇒ $7x - y = 9$

⇒ $4x + 3y = -2$

⇒ $21x - 3y = 27$

⇒ Adding: $25x = 25$;

⇒ $x = 1$ and $y = -2$

⇒ centre $(1, -2)$

⇒ So, centre $(1, -2)$



Radius is distance between (1,-2) and (5,1)

$$\Rightarrow r = \sqrt{(5-1)^2 + (1-(-2))^2}$$

$$\Rightarrow r = \sqrt{16+9}$$

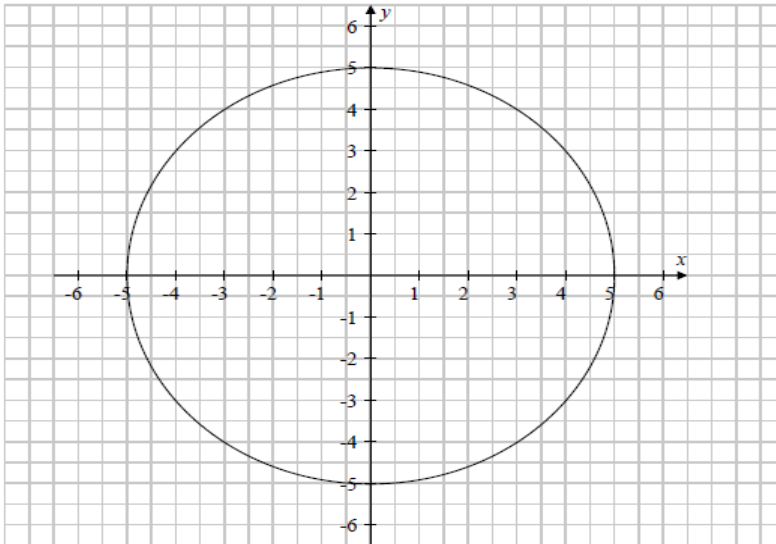
$$\Rightarrow r = 5$$

$$\Rightarrow \text{Equation of circle: } (x-1)^2 + (y-(-2))^2 = 5^2$$

$$\Rightarrow x^2 + y^2 - 2x + 4y - 20 = 0$$

Q7.

a)



(b) Verify, using algebra, that $A(-4, 3)$ is on c .

$$\begin{aligned} x^2 + y^2 &= 25 \\ (-4)^2 + 3^2 &= 16 + 9 = 25 = \text{RHS} \end{aligned}$$

or

$$\begin{aligned} \text{Centre of } c: O(0, 0) \\ |OA| &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-4-0)^2 + (3-0)^2} \\ &= \sqrt{25} = 5 = \text{radius of } c \end{aligned}$$

(c) Find the equation of the circle with centre $(-4, 3)$ that passes through the point $(3, 4)$.

$$\begin{aligned} r &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(3-(-4))^2 + (4-3)^2} = \sqrt{49+1} = \sqrt{50} \\ (x-h)^2 + (y-k)^2 &= r^2 \Rightarrow (x+4)^2 + (y-3)^2 = (\sqrt{50})^2 = 50 \end{aligned}$$



Q8.

a)

$$\sqrt{g^2 + f^2 - c} = \sqrt{1+1+7} = 3$$

(a) Complete the following table:

Circle	Centre	Radius	Equation
c_1	$(-3, -2)$	2	$(x+3)^2 + (y+2)^2 = 4$ OR $x^2 + y^2 + 6x + 4y + 9 = 0$
c_2	$(1, 1)$	3	$x^2 + y^2 - 2x - 2y - 7 = 0$

(b) (i) Find the co-ordinates of the point of contact of c_1 and c_2 .

Divide line segment joining $(-3, -2)$ and $(1, 1)$ in ratio 2 : 3

$$\left(\frac{2(1) + 3(-3)}{2+3}, \frac{2(1) + 3(-2)}{2+3} \right) = \left(-\frac{7}{5}, -\frac{4}{5} \right)$$

OR

$$\text{Slope line of centres} = \frac{3}{4}$$

$$\text{Equation line of centres: } y - 1 = \frac{3}{4}(x - 1) \Rightarrow 3x - 4y + 1 = 0$$

$$c_1 - c_2 = 4x + 3y + 8 = 0$$

$$4x + 3y + 8 = 0 \cap 3x - 4y + 1 = 0 \Rightarrow x = -\frac{7}{5}, y = -\frac{4}{5}$$



- (ii) Hence, or otherwise, find the equation of the tangent, t , common to c_1 and c_2 .

$$\text{Slope of line of centres: } \frac{1+2}{1+3} = \frac{3}{4}$$

$$\text{Slope of tangent: } m = -\frac{4}{3}$$

$$\begin{aligned} \text{Equation of tangent: } y + \frac{4}{5} &= -\frac{4}{3}\left(x + \frac{7}{5}\right) \\ &\Rightarrow 3y + \frac{12}{5} = -4x - \frac{28}{5} \\ &\Rightarrow 4x + 3y + 8 = 0 \end{aligned}$$

OR

$$\begin{aligned} c_1 - c_2 &= x^2 + y^2 + 6x + 4y + 9 - (x^2 + y^2 - 2x - 2y - 7) = 0 \\ &\Rightarrow 6x + 4y + 9 - (-2x - 2y - 7) = 0 \\ &\Rightarrow 8x + 6y + 16 = 0 \Rightarrow 4x + 3y + 8 = 0 \end{aligned}$$

OR

$$\begin{aligned} xx_1 + yy_1 + g(x+x_1) + f(y+y_1) + c &= 0 \\ x\left(-\frac{7}{5}\right) + y\left(-\frac{4}{5}\right) + 3\left(x + \left(-\frac{7}{5}\right)\right) + 2\left(y + \left(-\frac{4}{5}\right)\right) + 9 &= 0 \\ \Rightarrow 4x + 3y + 8 &= 0 \end{aligned}$$

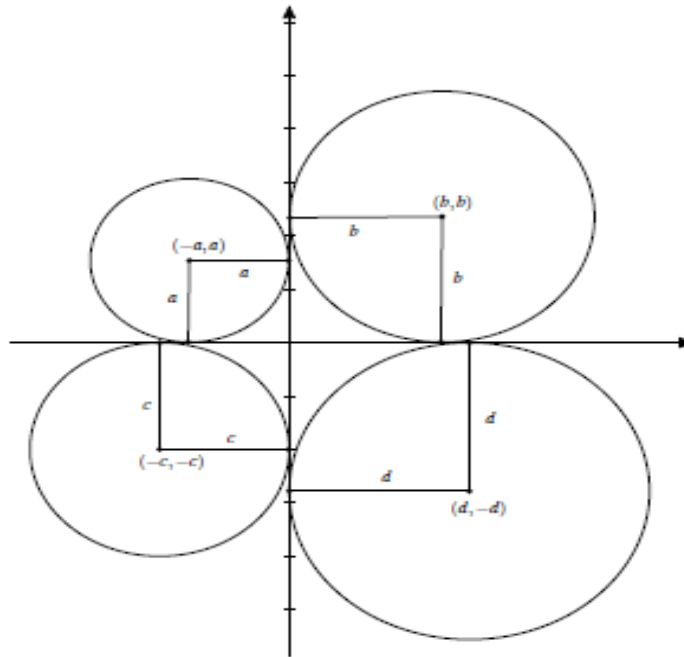


Q9.

a)

The centre of a circle lies on the line $x + 2y - 6 = 0$. The x -axis and the y -axis are tangents to the circle. There are two circles that satisfy these conditions. Find their equations.

Consider the diagram below:



Note that this diagram is not meant to represent the solution. It is just meant to illustrate the possibilities for circles that are tangent to both axes. In the diagram, a , b , c and d are the radii of the circles. We can see from this diagram that if (x, y) is the centre of a circle that has both the x -axis and the y -axis as tangents, then either

- Case 1: $y = x$
- Case 2: $y = -x$

In either case the radius is either x or $-x$. Since the radius is positive, we can say that the radius is $|x|$ in either case. ...



Case 1: $y = x$. We are also told that $x + 2y - 6 = 0$. Substituting x for y in the latter equation gives

$$x + 2x - 6 = 0 \Leftrightarrow 3x - 6 = 0 \Leftrightarrow x = 2.$$

Now $y = x$ so $y = 2$. Therefore the centre of the circle has co-ordinates $(2, 2)$ and the radius is 2. Therefore in this case the circle has equation

$$(x - 2)^2 + (y - 2)^2 = 4.$$

Case 2: $y = -x$. As before we use this to substitute $-x$ for y in the equation $x + 2y - 6 = 0$. This gives

$$x + 2(-x) - 6 = 0 \Leftrightarrow -x - 6 = 0 \Leftrightarrow x = -6.$$

It follows that $y = -(-6) = 6$. So in this case the centre has co-ordinates $(-6, 6)$ and the radius is 6. So this circle has equation

$$(x + 6)^2 + (y - 6)^2 = 36.$$