



Question 1

(i) Solve for x :

$$2(4 - 3x) + 12 = 7x - 5(2x - 7).$$

$$8 - 6x + 12 = 7x - 10x + 35$$

$$-15 = 3x$$

$$x = -5$$

(ii) Verify your answer to (i) above.

$$x = -5$$

$$2(4 - (-15)) + 12$$

$$38 + 12$$

$$50$$

$$7(-5) - 5(-10 - 7)$$

$$-35 + 85$$

$$50$$

$$[50 = 50]$$

Question 2

Solve the simultaneous equations:

$$x + y = 7$$

$$x^2 + y^2 = 25.$$

$$x = 7 - y$$

$$(7 - y)^2 + y^2 = 25$$

$$y^2 - 7y + 12 = 0$$

$$(y - 4)(y - 3) = 0$$

$$y = 4 \quad y = 3$$

$$x = 7 - 4 \quad x = 7 - 3$$

$$x = 3 \quad x = 4$$

$$(3, 4) \quad (4, 3)$$



Question 3

Simplify $\frac{x^2 - xy}{x^2 - y^2}$

Factorise the numerator (the top line of the equation) and denominator (bottom line of the equation- difference of two squares)

$$\frac{x(x - y)}{(x + y)(x - y)}$$

$$\frac{x\cancel{(x - y)}}{(x + y)\cancel{(x - y)}}$$

$$= \frac{x}{(x + y)}$$

Question 4

Express the following as a single fraction in its simplest form:

$$\frac{6y}{x(x+4y)} - \frac{3}{2x}$$

Step 1:

Find the common denominator by multiplying the bottom lines:

$$2x * x(x + 4y) = 2x^2(x+4y)$$

So $2x^2(x+4y)$ is our **common denominator**

Step 2:

Find the numerator by cross multiplying (top lines by bottom lines):

$$\begin{aligned} 6y * 2x - 3 * x(x+4y) &= 12xy - 3 * (x^2 + 4xy) \\ &= 12xy - 3x^2 - 12xy \\ &= -3x^2 \end{aligned}$$

So $-3x^2$ is our **numerator**

Step 3:

The answer is the numerator divided by the denominator:

$$\frac{6y}{x(x+4y)} - \frac{3}{2x} = \frac{\text{Numerator}}{\text{Denominator}}$$

$$= \frac{-3x^2}{2x^2(x+4y)}$$

$$= \frac{-3}{2(x+4y)}$$

$$= \frac{-3}{2x+8y}$$



Question 5

Solve the simultaneous equations:

$$x^2 + xy + 2y^2 = 4$$

Equation (1)

$$2x + 3y = -1.$$

Equation (2) Line

Find x in terms of y using the linear equation; Equation (2) $2x + 3y = -1$

$$x = \frac{-3y - 1}{2}$$

Substitute $x = \frac{-3y-1}{2}$ into Equation (1)

$$\left(\frac{-3y - 1}{2}\right)^2 + \left(\frac{-3y - 1}{2}\right)y + 2y^2 = 4$$

Multiply across by 4

$$(-3y - 1)^2 + (-3y - 1)2y + 8y^2 = 16$$

Expand the bracket and take the 16 over to the left hand side

$$9y^2 + 6y + 1 - 6y^2 - 2y + 8y^2 - 16 = 0$$

Group terms together

$$11y^2 + 4y - 15 = 0$$

$$(11y + 15)(y - 1) = 0$$

$$y = \frac{-15}{11} \text{ or } y = 1$$

Substitute $y = \frac{-15}{11}$ or $y = 1$ into Equation (2) to solve for x

$$2x + 3\left(\frac{-15}{11}\right) = -1$$

$$2x + \left(\frac{-45}{11}\right) = -1$$

$$2x = -1 + \frac{45}{11}$$

$$x = \frac{17}{11}$$

OR

$$2x + 3(1) = -1$$

$$2x + 3 = -1$$

$$2x = -4$$



$$x = -2$$

Give the answer matching the appropriate x and y values:

$$\text{Answer} = \left(\frac{17}{11}, \frac{-15}{11}\right) \text{ and } (-2, 1)$$

Question 6

Express the following as a single fraction in its simplest form:

$$\frac{x^2 + 4}{x^2 - 4} - \frac{x}{x + 2}$$

Hint: $x^2 - 4$ is the difference between two squares i.e. $(x)^2 - (2)^2 = (x + 2)(x - 2)$

Step 1:

Find the common denominator by multiplying the bottom lines:

So $(x^2 - 4)(x+2)$ is our **common denominator**

Step 2:

Find the numerator by cross multiplying (top lines by bottom lines):

$$(x^2 + 4)(x+2) - x(x^2 - 4) = (x+2) * [(x^2 + 4) - x(x-2)] \quad \dots \text{ because : } x^2 - 4 = (x + 2)(x - 2)$$

So $(x+2) * [(x^2 + 4) - x(x-2)]$ is our **numerator**

Step 3:

The answer is the numerator divided by the denominator:

$$\begin{aligned} \frac{x^2 + 4}{x^2 - 4} - \frac{x}{x + 2} &= \frac{\text{Numerator}}{\text{Denominator}} \\ &= \frac{(x+2) * [(x^2 + 4) - x(x-2)]}{(x^2 - 4)(x+2)} \\ &= \frac{(x^2 + 4) - x(x-2)}{(x^2 - 4)} \quad \dots(x+2) \text{ cancels} \\ &= \frac{(x^2 + 4) - x^2 + 2x}{(x^2 - 4)} \\ &= \frac{2x+4}{(x^2 - 4)} \\ &= \frac{2(x+2)}{(x+2)(x-2)} \quad \dots\text{difference of two squares} \\ &= \frac{2}{(x-2)} \end{aligned}$$



Question 7

Find the range of values of x for which $|x - 4| \geq 2$, where $x \in \mathbb{R}$.

Method 1:

Expand the bracket $x^2 - 8x + 16 \geq 4$

Take the 4 to the left hand side $x^2 - 8x + 12 \geq 0$

Solve for the roots of the equation $(x - 2)(x - 6) \geq 0$

$$x = 2$$

$$x = 6$$

Answer $x \leq 2$ or $x \geq 6$

Method 2:

Split into 2 separate equations: $+(x - 4) \geq 2$ or $-(x - 4) \geq 2$

Solve each equation separately:

$$x - 4 \geq 2$$

$$x - 4 + 4 \geq 2 + 4$$

$$x \geq 6$$

OR

$$-(x - 4) \geq 2$$

$$+(x - 4) \leq -2$$

$$x - 4 + 4 \leq -2 + 4$$

$$x \leq 2$$

Question 8

Find the set of all real values of x for which $2x^2 + x - 15 \geq 0$.

Step 1:

For inequalities, first set the equation = 0 and solve.

$$2x^2 + x - 15 = 0$$

$$(2x-5)(x+3) = 0$$

$$2x-5=0 \quad x+3=0$$

$$x=2.5 \quad x=-3$$

Step 2:

Then look at the sign in the inequality in the question.

If the sign is ≤ 0 we are "between the posts" which means the answer will be in the format of number $\leq x \leq$ number e.g. $-3 \leq x \leq 2.5$

If the sign is ≥ 0 we are "outside the posts" which means the answer will be in the format of



$x \leq \text{number}$ and $x \geq \text{number}$ e.g. $x \leq -3$ and $x \geq 2.5$

Step 3:

Therefore in this case the answer is

$x \leq -3$ and $x \geq 2.5$ (You can sub in values to the original question to check if your answer is correct!)

Question 9

$$\begin{aligned}
 x &= \sqrt{x+6} \\
 \Rightarrow x^2 &= x+6 \\
 \Rightarrow x^2 - x - 6 &= 0 \\
 \Rightarrow (x+2)(x-3) &= 0 \\
 \Rightarrow x &= -2, \quad x = 3 \\
 x = -2: \quad -2 &\neq \sqrt{-2+6} = \sqrt{4} = 2 \quad \times \\
 x = 3: \quad 3 &= \sqrt{3+6} = \sqrt{9} = 3 \quad \checkmark
 \end{aligned}$$

Question 10

Solve the following for x, y and z.

$$\begin{aligned}
 x + 2y - z &= 1 \\
 2x + y + z &= 4 \\
 x + 2y + z &= 2
 \end{aligned}$$

Solution:

Step 1:

Number each equation

$$\begin{aligned}
 x + 2y - z &= 1 \quad \dots\dots\dots (1) \\
 2x + y + z &= 4 \quad \dots\dots\dots (2) \\
 x + 2y + z &= 2 \quad \dots\dots\dots (3)
 \end{aligned}$$

Step 2:

Add two equations together to find equations (4) and (5):

$$\begin{aligned}
 x + 2y - z &= 1 \quad \dots\dots\dots (1) \\
 \underline{x + 2y + z} &= 2 \quad \dots\dots\dots (3) \\
 2x + 4y &= 3 \quad \dots\dots\dots (4)
 \end{aligned}$$

$$\begin{aligned}
 x + 2y - z &= 1 \quad \dots\dots\dots (1) \\
 \underline{2x + y + z} &= 4 \quad \dots\dots\dots (2) \\
 3x + 3y &= 5 \quad \dots\dots\dots (5)
 \end{aligned}$$

Step 3:

Solve equations (4) and (5):

$$\begin{aligned}
 2x + 4y &= 3 \quad \dots\dots\dots (4) \\
 3x + 3y &= 5 \quad \dots\dots\dots (5) \\
 \\ \\
 6x + 12y &= 9 \quad \dots\dots\dots (4) \quad (\times 3) \\
 6x + 6y &= 10 \quad \dots\dots\dots (5) \quad (\times 2) \\
 \\ \\
 6x + 12y &= 9 \quad \dots\dots\dots (4) \\
 \underline{-6x - 6y} &= -10 \quad \dots\dots\dots (5) \quad (\times -1) \\
 6y &= -1
 \end{aligned}$$



$$y = -1/6$$

$$2x+4y = 3 \quad (\text{equation 4})$$

$$2x+4(-1/6) = 3$$

$$x = 11/6$$

$$x + 2y - z = 1 \quad (\text{Equation 1})$$

$$(11/6) + 2(-1/6) - z = 1$$

$$z = 1/2$$

(You can sub in values to the original question to check if your answer is correct!)

Question 11

Solve the equation

$$|4x - 3| > 5$$

Solution 2:

Step 1:

Set the equation = instead of less than/greater than and solve

$$|4x - 3| = 5$$

This could mean that

$$4x - 3 = 5$$

or

$$4x - 3 = -5$$

$$4x = 5+3$$

$$4x = -5 + 3$$

$$4x = 8$$

$$4x = -2$$

$$x = 2$$

$$x = -1/2$$

Step 2:

Is the answer “between the roots” or “outside the roots”?

In this question the sign is > so the answer is outside the roots. So the answer is:

$$x > 2 \text{ and } x < -1/2$$

Question 12

Step 1:

Set the equation = instead of less than/greater than and solve

$$|3x + 2| < 4$$

This could mean that

$$3x + 2 = 4$$

or

$$3x + 2 = -4$$

$$3x = 4-2$$

$$3x = -4 -2$$

$$3x = 2$$

$$3x = -6$$

$$x = 2/3$$

$$x = -2$$

Step 2:

Is the answer “between the roots” or “outside the roots”?

In this question the sign is < so the answer is between the roots. So the answer is:

$$-2 < x < 2/3$$

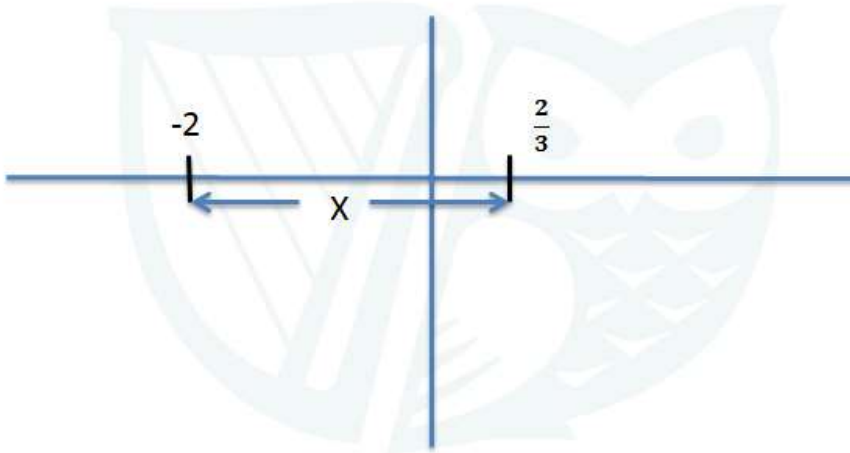
Step 3:

Is the answer between the roots or outside the roots?

(Note: that $x = -2$ and $x = 2/3$ should not be included on the number in the shaded region as inequality does not include equals, it's just less than)



This answer is between the roots so on a graph this looks like:



Question 13

Step 1:

Set the equation = 0

$$f(x) = 2x^3 - 4x^2 - 22x + 24 = 0$$

Step 2:

To find x, try a few values of x and see if they give you zero.

Try x=0 first:

$$\begin{aligned} f(0) &= 2 * 0 - 4 * 0 - 22 * 0 + 24 \\ &= 24 \neq 0 \end{aligned}$$

This is not equal to zero so x=0 is not a solution (or “root”).

Try x=1:

$$\begin{aligned} f(1) &= 2 * 1 - 4 * 1 - 22 * 1 + 24 \\ &= 0! \end{aligned}$$

This is equal to zero so **x=1** is a solution (or “root”) which means that **(x-1)** is a **factor**.

Step 3:

Divide the equation by the factor you just found.

$$\begin{array}{r} 2x^2 - 2x - 24 \\ (x-1) \overline{) 2x^3 - 4x^2 - 22x + 24} \\ \underline{2x^3 - 2x^2} \\ -2x^2 - 22x + 24 \\ \underline{-2x^2 + 2x} \\ -24x + 24 \\ \underline{-24x + 24} \\ 0 \end{array}$$

Step 4:



Factorise fully.

$$\begin{aligned} 2x^3 - 4x^2 - 22x + 24 &= (x-1)(2x^2 - 2x - 24) \\ &= (x-1)(2x+6)(x-4) \end{aligned}$$

Step 5:

Pull out the final answers, also called “roots”.

$$x-1 = 0$$

$$\mathbf{x=1}$$

$$2x+6 = 0$$

$$2x = -6$$

$$\mathbf{x = -3}$$

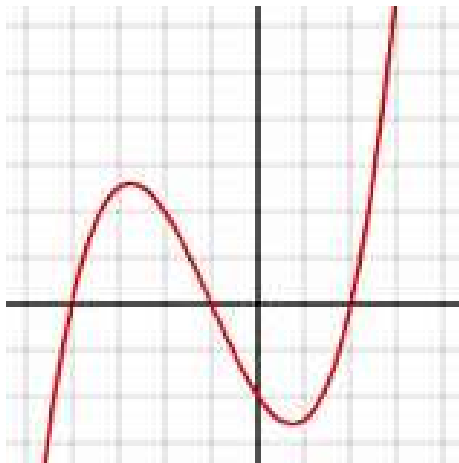
$$x-4 = 0$$

$$\mathbf{x = 4}$$

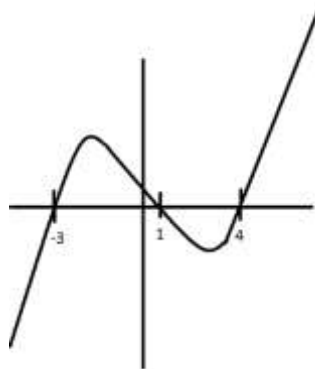
$x = -3, 1, 4$ are the solutions

Step 6:

The graph always looks something like this:



So just make sure it crosses the x axis at your solutions $x = -3, 1, 4$:





Question 14

(a)

$$x = -3, \quad x = -1, \quad x = 2$$

$$f(x) = (x+3)(x+1)(x-2) = x^3 + 2x^2 - 5x - 6$$

OR

$$f(x) = x^3 + 2x^2 - 5x - 6$$

$$f(-3) = -27 + 18 + 15 - 6 = 0 \Rightarrow (x+3) \text{ is a factor}$$

$$f(-1) = -1 + 2 + 5 - 6 = 0 \Rightarrow (x+1) \text{ is a factor}$$

$$f(2) = 8 + 8 - 10 - 6 = 0 \Rightarrow (x-2) \text{ is a factor}$$

$$f(x) = (x+3)(x+1)(x-2) = x^3 + 2x^2 - 5x - 6$$

(b) (i)

$$f(x) = g(x)$$

$$x^3 + 2x^2 - 5x - 6 = -2x - 6$$

$$\Rightarrow x^3 + 2x^2 - 3x = 0$$

$$\Rightarrow x(x^2 + 2x - 3) = 0$$

$$\Rightarrow x(x-1)(x+3) = 0$$

$$\Rightarrow x = 0, \quad x = 1, \quad x = -3$$

$$\Rightarrow y = -6, \quad y = -8, \quad y = 0$$

Points: $(-3, 0)$, $(0, -6)$, $(1, -8)$

(ii)

$$g(x) = -2x - 6$$

$$g(-3) = -2(-3) - 6 = 6 - 6 = 0 \Rightarrow (-3, 0)$$

$$g(0) = -2(0) - 6 = -6 \Rightarrow (0, -6)$$

Question 15

a (i)

$$f(x) = x^3 + kx^2 - 4x - 12$$

$$(x+3) \text{ is factor} \Rightarrow f(-3) = 0$$

$$f(-3) = (-3)^3 + k(-3)^2 - 4(-3) - 12 = 0$$

$$-27 + 9k + 12 - 12 = 0$$

$$9k = 27 \Rightarrow k = 3$$



(ii)

$$\begin{aligned} \frac{3}{1+x^p} + \frac{3}{1+x^{-p}} &= \frac{3(1+x^{-p}) + 3(1+x^p)}{(1+x^p)(1+x^{-p})} \\ &= \frac{3(1+x^{-p} + 1+x^p)}{1+x^p + x^{-p} + x^0} \\ &= \frac{3(2+x^{-p} + x^p)}{(2+x^{-p} + x^p)} \\ &= 3 \end{aligned}$$

Question 16

(i) Let x = Stage number.

There are 4 times as many blue tiles than the Stage number

Blue tiles = $4x$

There are 4 white tiles in every stage. This is a constant and remains 4 no matter what stage number we use.

The total number of green tiles is the square of the stage number.

Number of Green tiles = x^2

The total number of tiles (T) must be the green tiles + blue tiles + white tiles

$$T = x^2 + 4x + 4$$

(ii) $x^2 + 4x + 4 = 324$

Factorise $(x+2)(x+2) = 324$

$$(x+2)^2 = 324$$

$$x+2 = 18$$

$$x = 16$$

There are x^2 green tiles therefore $16^2 = 256$ green tiles.

(iii) Mary's kitchen is square. Therefore the length of each side = $\sqrt{6.76} = 2.6\text{m} = 260\text{ cm}$.

Each tile has sides of 20 cm each and $13 \times 20 = 260$. Therefore there are 13 tiles on each side or in each row.

In the first row there are two white tiles and the rest ($13-2=11$) are blue. Therefore this must be stage 11.



$$\text{Green} = x^2 = 121$$

$$\text{Blue} = 4x = 44$$

$$\text{White} = 4$$

Check: The total number of tiles = $121 + 44 + 4 = 169$.

The area of each tile = $0.20 \times 0.20 = 0.04 \text{ m}^2$. The total number of tiles needed = $6.76 \div 0.04 = 169$.