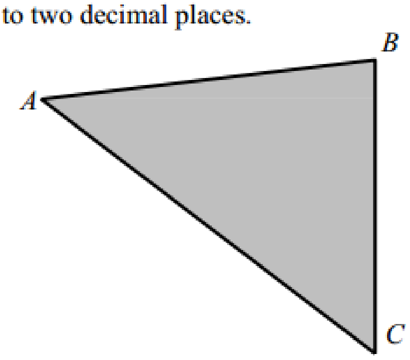
Trigonometery

Question 1.

Solution:

(a) (i) Find $|\angle CBA|$. Give your answer, in degrees, correct to two decimal places.

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$
$$= \frac{120^2 + 134^2 - 150^2}{2(120)(134)}$$
$$= \frac{9856}{32160}$$
$$= 0 \cdot 306468$$
$$\Rightarrow B = 72 \cdot 15°$$



Solution:

(ii) Find the area of the triangle ACB correct to the nearest whole number.

$$\text{Area } \Delta ABC = \tfrac{1}{2} ac \sin B = \tfrac{1}{2}(120)(134)\sin 72 \cdot 15$$
$$= 7652 \cdot 97$$
$$\approx 7653 \text{ m}^2$$

Question 2.

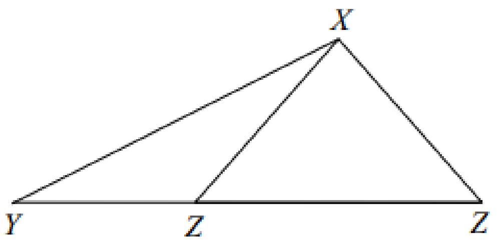
In a triangle XYZ, $|XY| = 5$ cm, $|XZ| = 3$ cm and $|\angle XYZ| = 27°$.

(i) Find the two possible values of $|\angle XZY|$. Give your answers correct to the nearest degree.

$$\frac{3}{\sin 27°} = \frac{5}{\sin \angle Z} \Rightarrow \sin \angle Z = \frac{5\sin 27°}{3} = 0\cdot 756$$

$$\Rightarrow |\angle Z| = 49° \quad \text{or} \quad |\angle Z| = 131°$$

(ii) Draw a sketch of the triangle XYZ, showing the two possible positions of the point Z.



(iii) solution:

In the case that $|\angle XZY| < 90°$, write down $|\angle ZXY|$, and hence find the area of the triangle XYZ, correct to the nearest integer.

$$|\angle ZXY| = 180° - (27° + 49°) = 104°$$

$$\Delta = \tfrac{1}{2}ab\sin C = \tfrac{1}{2}(5)(3)\sin 104° = 7\cdot 27 = 7\text{ cm}^2$$

Use 49° as 49°< 90°

- 27° is the angle at Y
- All angles must sum to 180° so the remaining angle must equal 104°

Question 3.

A glass Roof Lantern in the shape of a pyramid has a rectangular base *CDEF* and its apex is at *B* as shown. The vertical height of the pyramid is $|AB|$, where *A* is the point of intersection of the diagonals of the base as shown in the diagram.
Also $|CD| = 2{\cdot}5$ m and $|CF| = 3$ m.

(a) **(i)** Show that $|AC| = 1{\cdot}95$ m, correct to two decimal places.

(a) (i)

$$|EC|^2 = 3^2 + 2{\cdot}5^2 = 15{\cdot}25$$

$$|EC| = \sqrt{15{\cdot}25}$$

$$|EC| = 3{\cdot}905$$

$$\Rightarrow |AC| = 1{\cdot}9525$$

$$= 1{\cdot}95$$

(ii) The angle of elevation of *B* from *C* is 50° (i.e. $|\angle BCA| = 50°$).
Show that $|AB| = 2{\cdot}3$ m, correct to one decimal place.

(a) (ii)

$$\tan 50° = \frac{|AB|}{1{\cdot}95}$$

$$|AB| = 1{\cdot}95(1{\cdot}19175) = 2{\cdot}23239$$

$$|AB| = 2{\cdot}3$$

(iii) Find $|BC|$, correct to the nearest metre.

(a) (iii)

$$|BC|^2 = 1{\cdot}95^2 + 2{\cdot}3^2$$

$$|BC| = 3 \cdot 015377$$

$$|BC| = 3$$

Also: $\sin 40° = \dfrac{1{\cdot}95}{|BC|}$ **or** $\cos 40° = \dfrac{2{\cdot}3}{|BC|}$ **or**

$\cos 50° = \dfrac{1{\cdot}95}{|BC|}$ **or** $\sin 50° = \dfrac{2{\cdot}3}{|BC|}$

(iv) Find $|\angle BCD|$, correct to the nearest degree.

(a)
(iv)

$$3^2 = 3^2 + 2{\cdot}5^2 - 2(3)(2{\cdot}5)\cos\propto$$

$$15\cos\propto = 6{\cdot}25$$

$$\propto = 65°$$

or

$$\cos\propto = \frac{1{\cdot}25}{3}$$

$$\propto = 65°$$

(v) Find the area of glass required to glaze all four triangular sides of the pyramid. Give your answer correct to the nearest m^2.

(a)
(v)

$A = 2\times$ isosceles triangle $+ 2\times$ equilateral triangle

$$= 2\times\left[\tfrac{1}{2}(2{\cdot}5)(3)\sin 65°\right] +$$

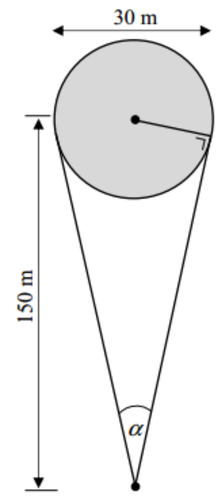
$$2\times\left[\frac{1}{2}(3)(3)\sin 60°\right]$$

$$= 14{\cdot}59$$

A=15

Question 4.

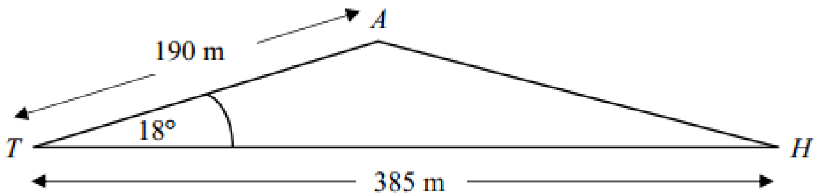
(a) Joan is playing golf. She is 150 m from the centre of a circular green of diameter 30 m. The diagram shows the range of directions in which Joan can hit the ball so that it could land on the green. Find α, the measure of the angle of this range of directions. Give your answer, in degrees, correct to one decimal place



Solution:

$$\sin \tfrac{1}{2}\alpha = \frac{15}{150} = 0 \cdot 1$$
$$\Rightarrow \tfrac{1}{2}\alpha = 5 \cdot 739°$$
$$\Rightarrow \alpha = 11 \cdot 478°$$
$$\alpha = 11 \cdot 5°$$

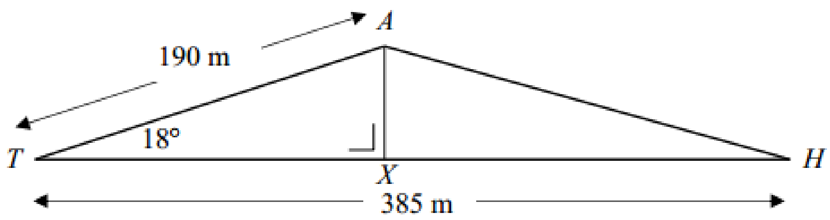
(b) At the next hole, Joan, at T, attempts to hit the ball in the direction of the hole H. Her shot is off target and the ball lands at A, a distance of 190 metres from T, where $|\angle ATH| = 18°$. $|TH|$ is 385 metres. Find $|AH|$, the distance from the ball to the hole, correct to the nearest metre.



Solution:

$$|AH|^2 = 190^2 + 385^2 - 2(190)(385)\cos 18°$$
$$= 36100 + 148225 - 139139 \cdot 5683$$
$$= 45185 \cdot 4317$$
$$|AH| = 212 \cdot 57 = 213$$

(b) alternative solution:



Draw AX perpendicular to TH

triangle ATX: $\sin 18° = \frac{|AX|}{190} \Rightarrow |AX| = 58 \cdot 71$

$\cos 18° = \frac{|TX|}{190} \Rightarrow |TX| = 180 \cdot 7$

$\Rightarrow |XH| = 204 \cdot 3$

$\Rightarrow |AH|^2 = (58 \cdot 71)^2 + (204 \cdot 3)^2$

$\Rightarrow |AH| = 212 \cdot 566 = 213$

Probability

Question 1 **(25 marks)**

(a) Explain each of the following terms:

(i) Sample space

> The set of all possible outcomes of an experiment.

(ii) Mutually exclusive events

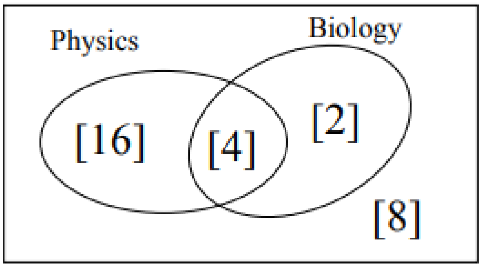
> Events E and F are mutually exclusive if they have no outcomes in common.
> i.e $P(E \cup F) = P(E)+P(F)$

(iii) Independent events

> Two events are independent if the outcome of one does not depend on the outcome of the other.
> i.e $P(E \cap F) = P(E).P(F)$ or $P(E|F) = P(E)$ or $P(F|E) = P(F)$

(b) In a class of 30 students, 20 study Physics, 6 study Biology and 4 study both Physics and Biology.

(i) Represent the information on the Venn Diagram.



A student is selected at random from this class.
The events E and F are:

E: The student studies Physics
F: The student studies Biology.

(ii) By calculating probabilities, investigate if the events E and F are independent.

> $P(E \cap F) = \frac{4}{30}$
>
> $P(E) \times P(F) = \frac{20}{30} \times \frac{6}{30} = \frac{4}{30}$
>
> $P(E \cap F) = P(E) \times P(F) \Rightarrow$ E and F are independent events

Question 6

Q1	Model Solution – 25 Marks	Marking Notes
(a)	$\frac{4}{5}\times\frac{1}{5}\times\frac{4}{5}\times\frac{1}{5}\times\frac{4}{5}\times\frac{1}{5}\times\frac{4}{5}=\frac{256}{78125}$ **or** $=0{\cdot}0032768$	**Scale 10C (0, 4, 5, 10)** *Low Partial Credit:* • $\frac{4}{5}$ • $(\frac{1}{5})^3$ *High Partial Credit:* • $\frac{4}{5}\times\frac{1}{5}\times\frac{4}{5}\times\frac{1}{5}\times\frac{4}{5}\times\frac{1}{5}\times\frac{4}{5}$ in any order
(b)	$\binom{6}{3}\left(\frac{1}{5}\right)^3\left(\frac{4}{5}\right)^3\left(\frac{1}{5}\right)$ $=\frac{1280}{78125}$ **or** $\frac{256}{15625}$ **or** $0{\cdot}016384$	**Scale 5D (0, 2, 3, 4, 5)** *Low Partial Credit:* • $\binom{6}{3}$ or $\left(\frac{1}{5}\right)^3$ or $\left(\frac{4}{5}\right)^3$ • $\frac{1}{5}$ for last day *Mid Partial Credit:* • $\binom{6}{3}\left(\frac{1}{5}\right)^3\left(\frac{4}{5}\right)^3$ and stops or continues • $\binom{7}{4}\left(\frac{1}{5}\right)^4\left(\frac{4}{5}\right)^3$ and continues *High Partial Credit:* • $\binom{6}{3}\left(\frac{1}{5}\right)^3\left(\frac{4}{5}\right)^3\left(\frac{1}{5}\right)$
(c)	$1-\left(\frac{4}{5}\right)^n$	**Scale 5B (0, 3, 5)** *Partial Credit:* • 1 or $\left(\frac{4}{5}\right)^n$ • any correct term from the expansion
(d)	$1-\left(\frac{4}{5}\right)^n>0{\cdot}99$ $\left(\frac{4}{5}\right)^n<0{\cdot}01$ $\left(\frac{4}{5}\right)^{20{\cdot}6377}\approx 0{\cdot}01000000517$ $n=21$	**Scale 5C (0, 2, 4, 5)** *Low Partial Credit:* • Ans **(c)** $>0{\cdot}99$ *High Partial Credit:* • viable solution to inequality • n = 20·6377 and stops

Question 7

Question 1 **(25 marks)**

An experiment consists of throwing two fair, standard, six-sided dice and noting the sum of the two numbers thrown. If the sum is 9 or greater it is recorded as a "win" (W). If the sum is 8 or less it is recorded as a "loss" (L).

(a) Complete the table below to show all possible outcomes of the experiment.

		Die 2					
		1	2	3	4	5	6
Die 1	1	L	L	L	L	L	L
	2	L	L	L	L	L	L
	3	L	L	L	L	L	W
	4	L	L	L	L	W	W
	5	L	L	L	W	W	W
	6	L	L	W	W	W	W

(b) **(i)** Find the probability of a win on one throw of the two dice.

$\frac{10}{36} = \frac{5}{18}$

(ii) Find the probability that each of 3 successive throws of the two dice results in a loss. Give your answer correct to four decimal places.

$\left(\frac{13}{18}\right)^3 = 0.3767$

(c) The experiment is repeated until a total of 3 wins occur. Find the probability that the third win occurs on the tenth throw of the two dice. Give your answer correct to four decimal places.

2 wins from 9 throws

$n = 9$
$r = 2$
$p = 5/18, \quad q = 13/18$

$P(X=r) = \binom{n}{r} p^r q^{n-r}$

$p(X=2) = \binom{9}{2}\left(\frac{5}{18}\right)^2\left(\frac{13}{18}\right)^7$

2 wins from 9 throws and a win on the tenth throw

$= \binom{9}{2}\left(\frac{5}{18}\right)^2\left(\frac{13}{18}\right)^7\left(\frac{5}{18}\right) = \binom{9}{2}\left(\frac{5}{18}\right)^3\left(\frac{13}{18}\right)^7 = 0.07908$

$= 0.0791$

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Question 8

Question 8 **(65 marks)**

In basketball, players often have to take free throws. When Michael takes his first free throw in any game, the probability that he is successful is 0·7.
For all subsequent free throws in the game, the probability that he is successful is:

- 0·8 if he has been successful on the previous throw
- 0·6 if he has been unsuccessful on the previous throw.

(a) Find the probability that Michael is successful (S) with all three of his first three free throws in a game.

$$P(S, S, S) = 0 \cdot 7 \times 0 \cdot 8 \times 0 \cdot 8 = 0 \cdot 448$$

(b) Find the probability that Michael is unsuccessful (U) with his first two free throws and successful with the third.

$$P(U, U, S) = 0 \cdot 3 \times 0 \cdot 4 \times 0 \cdot 6 = 0 \cdot 072$$

(c) List all the ways that Michael could be successful with his third free throw in a game and hence find the probability that Michael is successful with his third free throw.

S, S, S U, U, S S, U, S U, S, S

$$P(S, S, S) = 0 \cdot 7 \times 0 \cdot 8 \times 0 \cdot 8 = 0 \cdot 448$$
$$P(U, U, S) = 0 \cdot 3 \times 0 \cdot 4 \times 0 \cdot 6 = 0 \cdot 072$$
$$P(S, U, S) = 0 \cdot 7 \times 0 \cdot 2 \times 0 \cdot 6 = 0 \cdot 084$$
$$P(U, S, S) = 0 \cdot 3 \times 0 \cdot 6 \times 0 \cdot 8 = 0 \cdot 144$$

$$P = 0 \cdot 448 + 0 \cdot 072 + 0 \cdot 084 + 0 \cdot 144 = 0 \cdot 748$$

(d) **(i)** Let p_n be the probability that Michael is successful with his n^{th} free throw in the game (and hence $(1-p_n)$ is the probability that Michael is unsuccessful with his n^{th} free throw)
Show that $p_{n+1}=0{\cdot}6+0{\cdot}2p_n$.

$$\begin{aligned} p_{n+1} &= \mathrm{P(S,S)}+\mathrm{P(U,S)} \\ &= p_n\times 0{\cdot}8+(1-p_n)0{\cdot}6 \\ &= 0{\cdot}6+0{\cdot}2p_n \end{aligned}$$

(ii) Assume that p is Michael's success rate in the long run; that is, for large values of n, we have $p_{n+1}\approx p_n\approx p$.
Using the result from part **(d) (i)** above, or otherwise, show that $p=0{\cdot}75$.

$$\begin{aligned} & p\approx p_n\approx p_{n+1}=0{\cdot}6+0{\cdot}2p_n \\ & \Rightarrow 0{\cdot}8p_n=0{\cdot}6 \\ & \Rightarrow p_n=\frac{0{\cdot}6}{0{\cdot}8}=0{\cdot}75=p \end{aligned}$$

(e) For all positive integers n, let $a_n=p-p_n$, where $p=0{\cdot}75$ as above.

(i) Use the ratio $\frac{a_{n+1}}{a_n}$ to show that a_n is a geometric sequence with common ratio $\frac{1}{5}$.

$$\begin{aligned} \frac{a_{n+1}}{a_n} &= \frac{p-p_{n+1}}{p-p_n} \\ &= \frac{0{\cdot}75-(0{\cdot}6+0{\cdot}2p_n)}{0{\cdot}75-p_n} \\ &= \frac{0{\cdot}15-0{\cdot}2p_n}{5(0{\cdot}15-0{\cdot}2p_n)}=\frac{1}{5} \end{aligned}$$

(ii) Find the smallest value of n for which $p - p_n < 0{\cdot}00001$.

$a_n = p - p_n$
$a_1 = p - p_1 = 0\cdot75 - 0\cdot7 = 0\cdot05$

$ar^{n-1} = 0\cdot05(0\cdot2)^{n-1} < 0\cdot00001$
$(n-1)\ln 0\cdot2 < \ln 0\cdot0002$
$\Rightarrow n-1 > \dfrac{\ln 0\cdot0002}{\ln 0\cdot2} = 5\cdot29$
$\Rightarrow n > 6\cdot29$

$n = 7$

(f) You arrive at a game in which Michael is playing. You know that he has already taken many free throws, but you do not know what pattern of success he has had.

(i) Based on this knowledge, what is your estimate of the probability that Michael will be successful with his next free throw in the game?

Answer: $0\cdot75$ **or** p

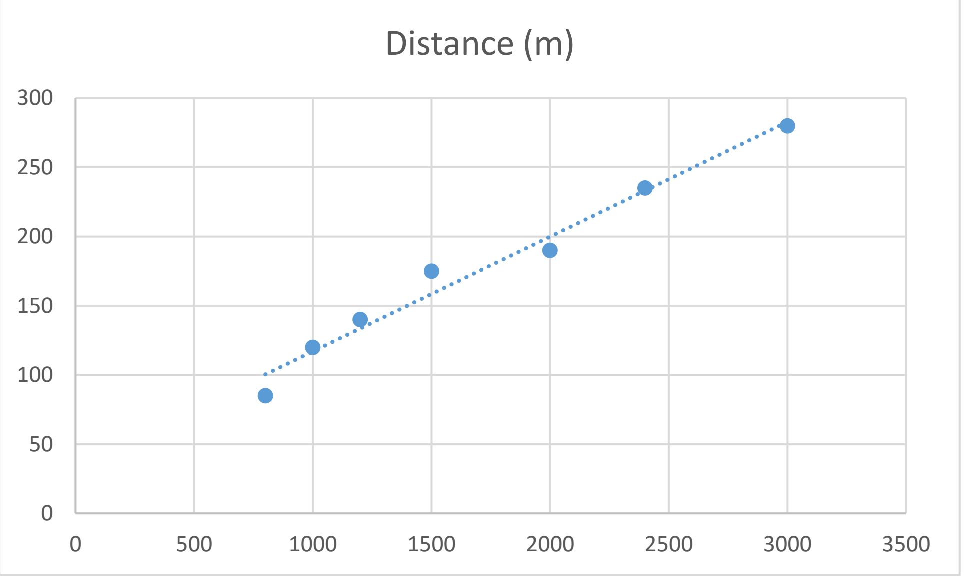
(ii) Why would it **not** be appropriate to consider Michael's subsequent free throws in the game as a sequence of Bernoulli trials?

Events not independent

Statistics

Question 9

i)



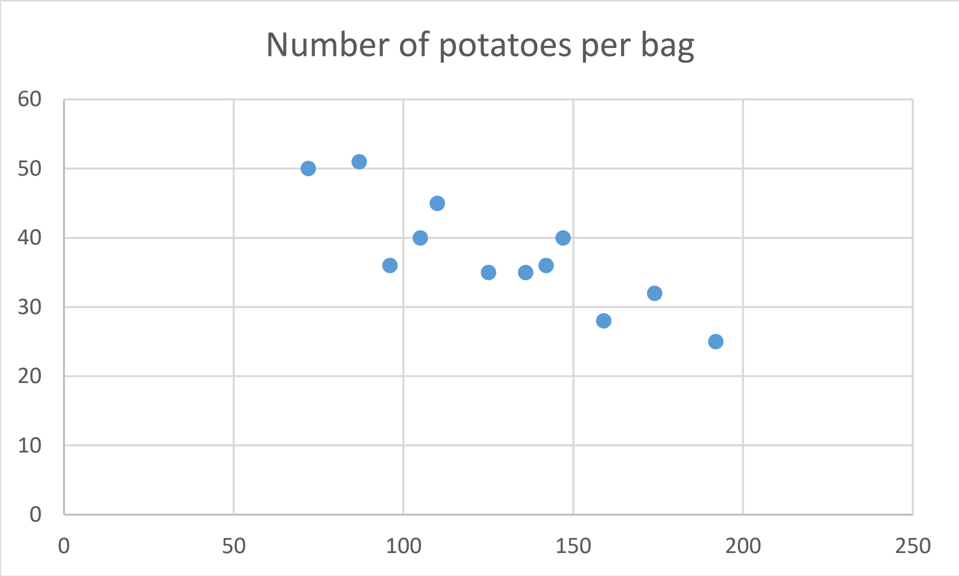
Mean engine capacity = 1,700, mean distance = 175

ii) Using the graph draw a line up at 1,800 cc and read the distance – should be approx. 184m and similarly for 125m should give a cc of approx. 1,050 cc (any reasonable answers here will be allowed once correct method is followed).

iii) r = 0.9875 using Pearson Correlation Coefficient

Question 10

i)



ii) c -negative correlation as line of best fit is downward sloping

iii) Accept the null hyptothesis as linear result from graph

iv) Correlation co-efficient is -0.85

v) This is close to 1 so can accept the null hypothesis