



Warm up questions

Question 1.

$$F = P(1+i)^t$$

$$F = (4000)(1+0.03)^4$$

$$F = \text{€}4,502.04$$

$$\text{Interest} = \text{€}4,502.04 - \text{€}4,000 = \text{€}502.04$$

Question 2.

$$F = P(1+i)^t$$

$$F = 1.12P$$

$$1.12P = P(1+i)^5$$

$$1.12 = (1+i)^5$$

$$1+i = \sqrt[5]{1.12}$$

$$1+i = 1.02292$$

$$i = 0.02292 = 2.3\% = \text{AER correct to 1 decimal place}$$

Rule for rounding “correct to 1 dp” is round to nearest number ?

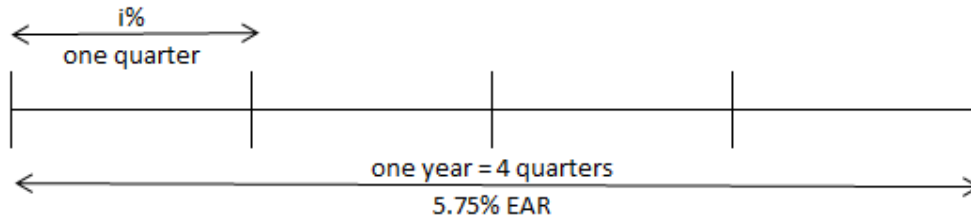
Question 3.

$$\frac{2500}{(1+0.03)^7} = 2032.73$$



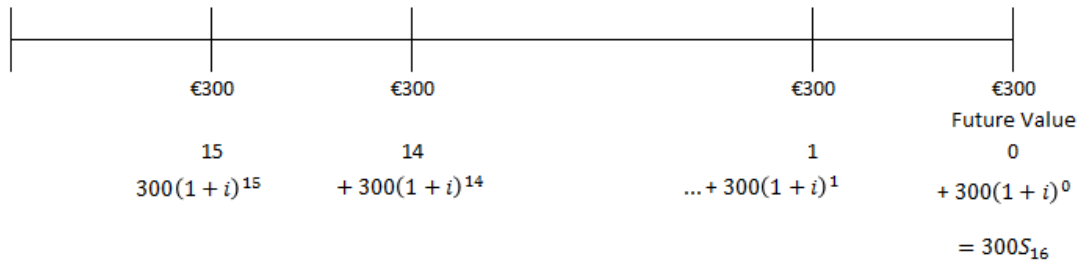
Compound Interest

Question 4. |



$$(1 + i)^4 = \left(1 + \frac{5.75}{100}\right)^1$$

$$1 + i = 1.0575^{\frac{1}{4}}$$



$$S_n = \frac{a(1 - r^n)}{1 - r}$$

$$a = 1$$

$$r = 1 + i$$

So, from above, $r = 1.0575^{\frac{1}{4}}$

$$\text{Hence, } S_{16} = \frac{1 \left(1 - \left(1.0575^{\frac{1}{4}}\right)^{16}\right)}{1 - 1.0575^{\frac{1}{4}}} = \frac{1 - 1.0575^4}{1 - 1.0575^{0.25}} = 17.80519556$$

Investment is worth $300S_{16} = 300 \times 17.80519556 = 5341.558669$
 $\approx \text{€}5341.56$

(Sense check: €300 per quarter = €1200 per year = €4800 in 4 years, plus interest seems ok.)



Question 5.

$$P(1 + i)^t = F$$

$$9950(1 + i)^5 = 12700$$

$$(1 + i)^5 = \frac{12700}{9950}$$

$$[(1 + i)^5]^{\frac{1}{5}} = \left[\frac{12700}{9950} \right]^{\frac{1}{5}}$$

$$1 + i = 1.050016511$$

$$i = 0.050016511 = 5.0016511\%$$

$$i \approx 5.00\%$$

Question 6.

Hint: The unknown (t) is the exponent. So, this involves using logs. Natural logs are easiest on the calculator.

$$P(1 + i)^t = F$$

$$10000(1.05)^t = 13000$$

$$1.05^t = \frac{13000}{10000} = 1.3$$

$$\ln(1.05^t) = \ln 1.3$$

$$t \ln 1.05 = \ln 1.3$$

$$t = \frac{\ln 1.3}{\ln 1.05} = 5.377400731 \text{ years}$$

$$t = 5.377400731 \times 12 = 64.52880877 \text{ months}$$

$$t \approx 65 \text{ months}$$



Question 7.

(i)

$$F = P(1+i)^t = 1(1+0.0035)^{12} = 1.042818$$

$$\Rightarrow i = 4.28\%$$

(ii)

$$F = P(1+i)^t$$

$$1.045 = 1(1+i)^{12}$$

$$1+i = \sqrt[12]{1.045} = 1.0036748$$

$$\Rightarrow i = 0.367\%$$

(iii) source of formula ?

$$A = P \left[\frac{i(1+i)^t}{(1+i)^t - 1} \right]$$

$$= 80000 \left[\frac{0.0035(1.0035)^{120}}{(1.0035)^{120} - 1} \right]$$

$$= 80000 \left[\frac{0.005322296}{0.520846} \right]$$

$$= 817.59 = \text{€}818$$

or

$$80000 = \frac{A}{1.0035} + \frac{A}{1.0035^2} + \dots + \frac{A}{1.0035^{120}}$$

$$= A \left[\frac{1}{1.0035} + \frac{1}{1.0035^2} + \dots + \frac{1}{1.0035^{120}} \right]$$

$$= A \left[\frac{\frac{1}{1.0035} \left(1 - \left(\frac{1}{1.0035} \right)^{120} \right)}{1 - \frac{1}{1.0035}} \right]$$

$$= A \left[\frac{0.342471198}{0.0035} \right]$$

$$= A [97.8489137]$$

$$A = 817.58 = \text{€}818$$



Present Value

Question 8.

(i)

$$(1+i)^{12} = 1.04 \Rightarrow 1+i = \sqrt[12]{1.04} = 1.003273 \Rightarrow i = 0.003274$$

Hence, $i = 0.327\%$

Or

$$\begin{aligned} (1.00327)^{12} &= 1.039953481 \\ &= 1.0400 \end{aligned}$$

$$r = 4\%$$

Where i is the monthly rate of interest and r is the annual equivalent rate.

(ii) “(ii)” should be above box

$$\begin{aligned} 15000 &= P(1.00327^{36} + 1.00327^{35} + \dots + 1.00327^2 + 1.00327) \\ \Rightarrow P \left[\frac{1.00327(1.00327^{36} - 1)}{1.00327 - 1} \right] &= 15000 \\ \Rightarrow P[38.26326387] &= 15000 \\ \Rightarrow P = 392.02 &= \text{€}392 \end{aligned}$$

(iii)

$$\begin{aligned} A &= P \frac{i(1+i)^t}{(1+i)^t - 1} \\ &= 15000 \left[\frac{0.00866(1+0.00866)^{36}}{1.00866^{36} - 1} \right] \\ &= 486.77 \\ \text{Monthly payment} & \text{€}487 \end{aligned}$$



Question 9. d

$$(i) \quad F = P(1 - i)^t$$

$$F = (20000)(1 - 0.15)^2$$

$$F = 20000 (0.85)^2$$

$$F = \text{€}14,450$$

$$(ii) \quad F = P(1 - i)^t$$

$$F = 14450(1 - 0.1)^3$$

$$F = 14450(0.9)^3$$

$$F = \text{€}10,534.05$$

Question 10. D

Year	Project X	Project Y
0	PV = -5000	PV = -3000
1	$PV = \frac{-2000}{1.05^1} = -1904.76$	$PV = \frac{-1000}{1.05^1} = -952.38$
2	$PV = \frac{2000}{1.05^2} = 1814.06$	$PV = \frac{2000}{1.05^2} = 1814.06$
3	$PV = \frac{5000}{1.05^3} = 4319.19$	$PV = \frac{3000}{1.05^3} = 2591.51$
NPV	-771.51	453.19
Advice	Reject	Invest

Amortisation Questions

Question 11. (B)

(i)

$$2.5\% \times 5000 = 125$$

(ii) To calculate monthly rate “i”



$$(1 + i)^{\frac{1}{12}} = (1.2175)^{\frac{1}{12}} = 1.016535$$

$$\text{Rate} = 1.65\%$$

(iii)

Payment number	Fixed monthly payment, €A	€A		New balance of debt (€)
		Interest	Previous balance reduced by (€)	
0				5000
1	125	82.50	42.50	4957.50
2	125	81.80	43.20	4914.30
3	125	81.09	43.91	4870.39

(iv) Source of formulae? Log Tables page ref ? I see ref to page 31 in Qs 12



$$A = p \left[\frac{i(1+i)^t}{(1+i)^t - 1} \right]$$

$$A[(1+i)^t - 1] = pi(1+i)^t$$

$$A(1+i)^t - A = pi(1+i)^t$$

$$A = (1+i)^t[A - pi]$$

$$\frac{A}{A - pi} = (1+i)^t$$

$$\frac{125}{125 - 5000\left(\frac{1.65}{100}\right)} = \left(1 + \frac{1.65}{100}\right)^t$$

$$\frac{125}{42.5} = (1.0165)^t$$

$$\log\left(\frac{125}{42.5}\right) = t \log(1.0165)$$

$$t = \frac{\log\left(\frac{125}{42.5}\right)}{\log(1.0165)}$$

$$t = 65.920$$

$$t = 66 \text{ months}$$

or

$$A = p \left[\frac{i(1+i)^t}{(1+i)^t - 1} \right]$$

$$125 = \frac{5000(0.0165)(1.0165)^t}{(1.0165)^t - 1}$$

$$125 = \frac{82.5(1.0165)^t}{(1.0165)^t - 1}$$

$$\frac{125}{82.5} = \frac{1.0165^t}{1.0165^t - 1}$$

$$\frac{50}{33} = \frac{1.0165^t}{1.0165^t - 1}$$

$$50(1.0165^t - 1) = 33(1.0165^t)$$

$$50(1.0165^t) - 50 = 33(1.0165^t)$$

$$50(1.0165^t) - 33(1.0165^t) = 50$$

$$1.0165^t(50 - 33) = 50$$

$$1.0165^t(17) = 50$$

$$1.0165^t = \frac{50}{17}$$

$$t \log 1.0165 = \log \frac{50}{17}$$

$$t = \frac{\log\left(\frac{50}{17}\right)}{\log 1.0165} = 65.92$$

$$t = 66 \text{ months}$$



(v)

$$\begin{aligned} A &= \frac{pi(1+i)^t}{(1+i)^t - 1} \\ &= \frac{5000 \left(1.085^{\frac{1}{52}} - 1 \right) (1.085)^3}{(1.085)^3 - 1} \\ &= \text{€}36.16 \end{aligned}$$

OR

Weekly interest rate $(1+i)^{52} = 1.085$

$$1+i = 1.085^{\frac{1}{52}}$$

$$1+i = 1.00157$$

$$i = 0.00157$$

$$A = \frac{pi(1+i)^t}{(1+i)^t - 1}$$

$$\begin{aligned} A &= \frac{5000(0.00157)(1.00157)^{156}}{(1.00157)^{156} - 1} \\ &= \text{€}36.16 \end{aligned}$$

(vi)

$$\begin{aligned} &125 \times 66 - (36.16)(156) \\ &= \text{€}2609.04 \end{aligned}$$



Question 12.

- (i) We can use the formula on page 31 as the interest rate and the compounding periods match; i.e. both monthly. (The formula book says “Annual” but we can replace this with “Monthly”)

$$A = P \frac{i(1+i)^t}{(1+i)^t - 1}$$

$$A = 5000 \frac{(0.01)(1.01)^6}{(1.01)^6 - 1}$$

$$A = 862.7418336 \approx \text{€}862.74 \text{ p.m.}$$

Monthly payment = €862.74 = (portion to pay interest) + (portion to reduce debt)

- (ii)

A	B	C	D = 1% x B	E = C-D	F=B-E
Month	Outstanding Balance at start month	Monthly Payment	Interest	Reduction in Debt	Outstanding Balance at end month
1	5000	862.74	50.00	812.74	4187.26
2	4187.26	862.74	41.87	820.87	3366.39
3	3366.39	862.74	33.66	829.08	2537.31
4	2537.31	862.74	25.37	837.37	1699.94
5	1699.94	862.74	17.00	845.74	854.20
6	854.20	862.74	8.54	854.20	0.00



Question 13

Solution Part (i)

At the end of the first year Tom will save 15% of his salary $60,000 * 15\% = 9,000$
 This will accumulate for 19 years at 3% pa $9,000 * [(1.03)^{19}]$
 Tom's contribution will increase in the second year (month 24) to $9,000 * 1.04 = 9,360$
 This will accumulate for 18 years at 3% pa $9,360 * [(1.03)^{18}]$
 The final contribution will be paid at the end of year 20 $9,000 * [1.04^{19}] = 18,961$

So Accumulated Amount will be

$$\begin{aligned}
 & 9,000 (1.03^{19} + 1.04 * (1.03^{18}) + \dots + 1.04^{19}] \\
 = & 9,000 * [(1.04^{19})] * [\{ (1.03/1.04) \}^{19} + \{ (1.03/1.04) \}^{18} + \dots + 1] \\
 = & 9,000 * 2.106849 * (1+i)^* [\{ 1 - (1/(1+i))^{20} \} / i] \quad \text{where } 1+i = (1.04/1.03) = 1.009709 \\
 = & 18,961 * 1.009709 * [1 - ((1/1.009709)^{20}) / 0.009709] = 19,145 * 18.09858 = 346,499
 \end{aligned}$$

Question 13

Solution Part (ii)

The accumulated value in part (i) equals the present value of a payment of 10,000 per annum , increasing at 5% per annum payable for n years. So

$$346,499 = 10,000 [1 + 1.05/1.03 + (1.05/1.03)^2 + \dots + (1.05/1.03)^{(n-1)}]$$

$$34.6499 = [1 + (1+i) + (1+i)^2 + \dots + (1+i)^{(n-1)}] \text{ where } (1+i) = (1.05/1.03) = 1.019417$$

$$34.6499 = [\{ (1+i)^n - 1 \} / i] = [\{ 1.019417^n - 1 \} / 0.019417]$$

$$0.672814 = \{ 1.019417^n - 1 \} \Rightarrow \ln 1.672814 = n \ln 1.019417 \Rightarrow n = 26.75$$

So the accumulated fund will be sufficient to pay the annuity for 26 years with three quarters of a years' payment remaining in the fund.