Society of Actuaries in Ireland

Probability Tutorial Questions 2020/2021

Question 1

[2016 Paper 2, Q5 (a)] (Part of a Past Exam Question worth 25 marks)

In an archery competition, the team consisting of John, David and Mike will win 1st prize if at least two of them hit the bullseye with their last arrows. From past experience, they know that the probability that John, David and Mike will hit the bullseye on their last arrow is 1/5, 1/6 and ¼ respectively.

i. Complete the table below to show all the ways in which they could win 1st prize.

	Way 1	Way 2	Way 3	Way 4
John	√			
David	√			
Mike	×			

Where

√=Hit

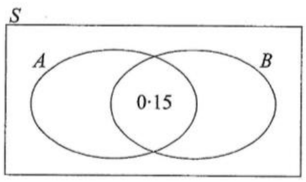
×=Miss

ii. Hence or otherwise find the probability that they will win the competition.

Question 2

[2010 SEC Paper 2, Q1] (25 marks)

Two events are such that P (A) =0.2, P(A ∩ B) =0.15 and P(A' ∩ B)=0.6



i) Find the probability that neither A nor B happens
ii) Find the conditional probability P(A|B)
iii) State whether A and B are independent events and justify your answer

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Question 3

[2012 SEC, Paper 2, Q4] (25 marks)

A certain basketball player scores 60% of the free-throw shots she attempts. During a particular game, she gets six free throws.

a) What assumption(s) can be made in order to regard this as a sequence of Bernoulli trials?
b) Based on such assumption(s), find, correct to three decimal places, the probability that:
   i. She scores on exactly four of the six shots
   ii. She scores for the second time on the fifth shot.

Question 4

[2018, Paper 2, Q1] (25 marks)

In a competition Mary has a probability of 1/20 of winning, a probability of 1/10 of finishing in second place, and a probability of 1/4 of finishing in third place. If she wins the competition she gets €9000. If she comes second she gets €7000 and if she comes third she gets €3000. In all other cases she gets nothing. Each participant in the competition must pay €2000 to enter.

(a) Find the expected value of Mary's loss if she enters the competition.

(b) Each of the 3 prizes in the competition above is increased by the same amount (€x) but the entry fee is unchanged. For example, if Mary wins the competition now, she would get €(9000 + x). Mary now expects to break even. Find the value of x.

Question 5

[2018, Paper 2, Q3] (25 marks)

(a) A security code consists of six digits chosen at random from the digits 0 to 9. The code may begin with zero and digits may be repeated. For example 0 7 1 7 3 7 is a valid code.

(i) Find how many of the possible codes will end with a zero.

(ii) Find how many of the possible codes will contain the digits 2 0 1 8 together and in this order.

(b) Find a, b, c and d if $\frac{(n+3)!(n+2)!}{(n+1)!(n+1)!} = an^3 + bn^2 + cn + d$, where a, b, c and d $\in \mathbb{N}$.

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Question 6

[2017, Paper 2, Q8 (b)] (Part of a Past Exam Question worth 60 marks)

In Galway, rain falls in the morning on 1/3 of the school days in the year.

When it is raining the probability of heavy traffic is ½.

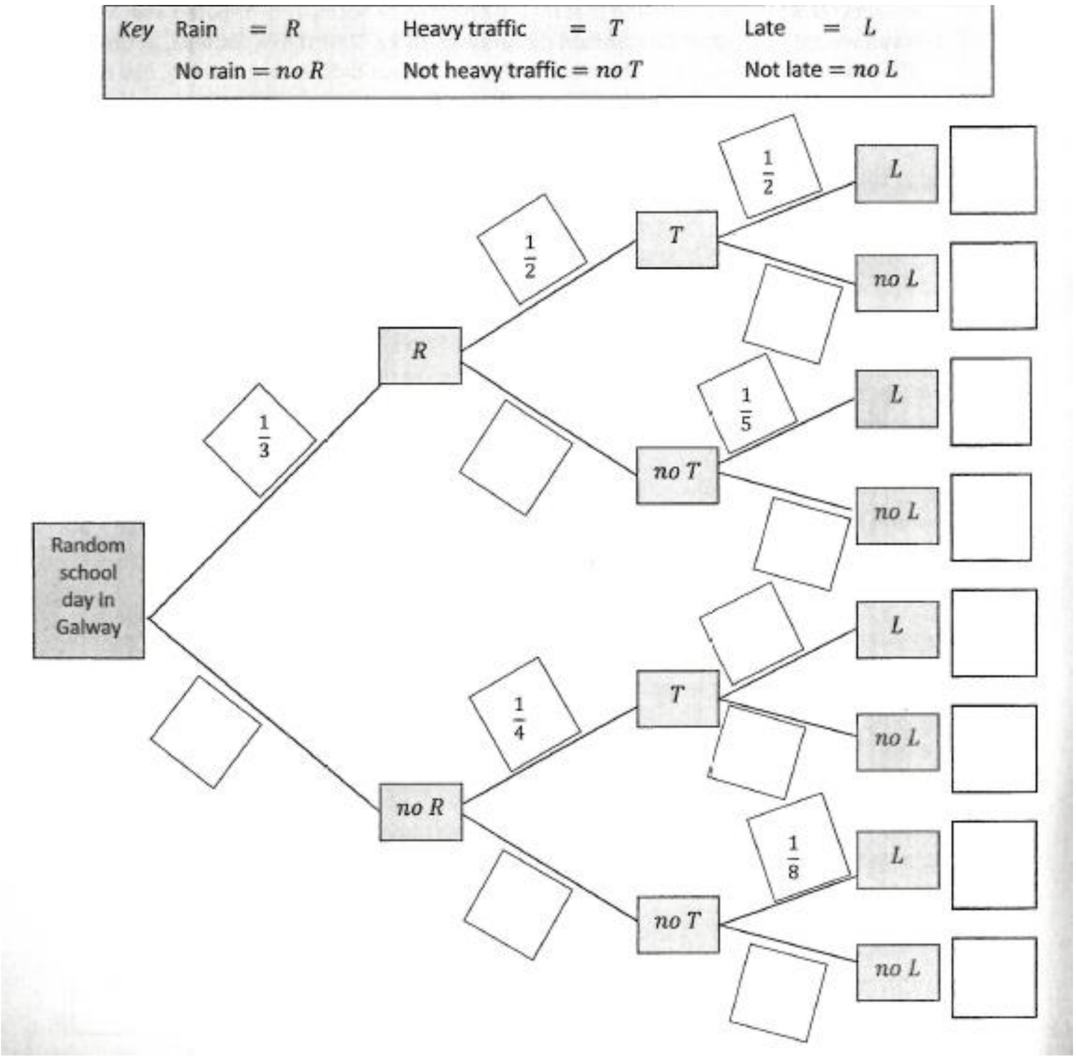
When it is not raining, the probability of heavy traffic is ¼.

When it is raining and there is heavy traffic, the probability of being late for school is ½.

When it is not raining and there is no heavy traffic, the probability of being late for school is 1/8.

In any other situation the probability of being late for school is 1/5.

Some of this information is shown in the tree diagram below.



i) Write the probability associated with each branch of the tree diagram **and** the probability of each outcome into the blank boxes provided.
Give each answer in the form of a/b, where a, b are natural numbers.

ii) On a random school day in Galway, find the probability of being late for school.

iii) On a random school day in Galway, find the probability that it rained in the morning, given you were late for school.

Question 7 **2013 Paper 2 Q2**

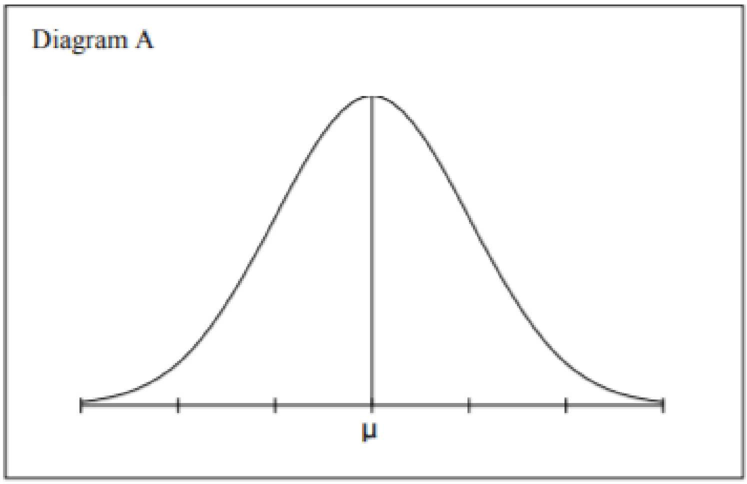
(a) A random variable X follows a normal distribution with mean 60 and standard deviation 5.

(i) Find $P(X \leq 68)$.

(ii) Find $P(52 \leq X \leq 68)$

(b) The heights of a certain type of plant, when ready to harvest, are known to be normally distributed, with a mean of μ. A company tests the effects of three different types of growth hormone on this type of plant. The three hormones were used on different large samples of the crop. After applying each hormone, it was found that the heights of the plants in the samples were still normally distributed at harvest time. The diagrams A, B and C show the expected distribution of the heights of the plants, at harvest time, without the use of the hormones. The effect, on plant growth, of each of the hormones is described. ***Sketch, on each diagram, a new distribution to show the effect of the hormone.***

Hormone A The effect of hormone A was to increase the height of all of the plants.



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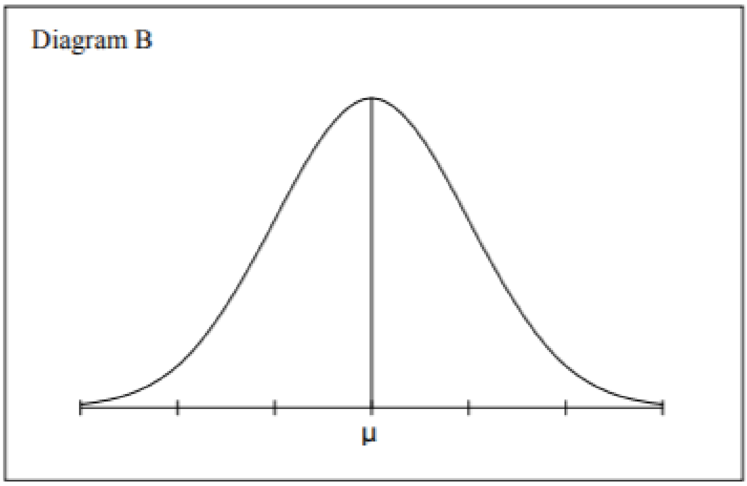
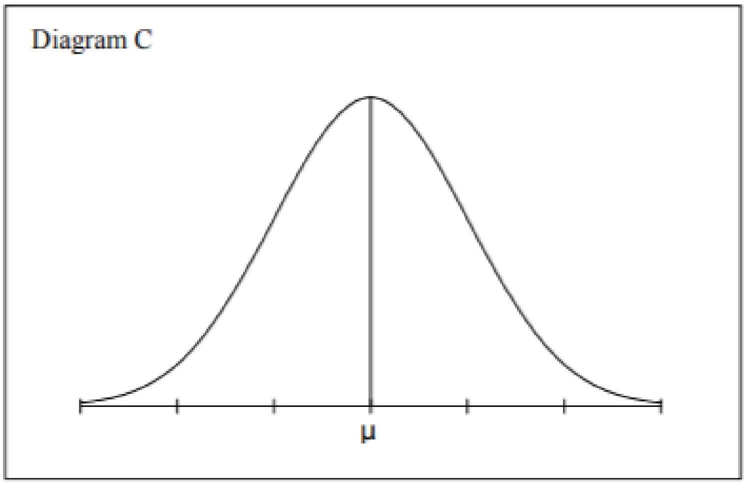
Hormone B The effect of hormone B was to reduce the number of really small plants and the number of really tall plants. The mean was unchanged.



Hormone C The effect of hormone C was to increase the number of small plants and the number of tall plants. The mean was unchanged.



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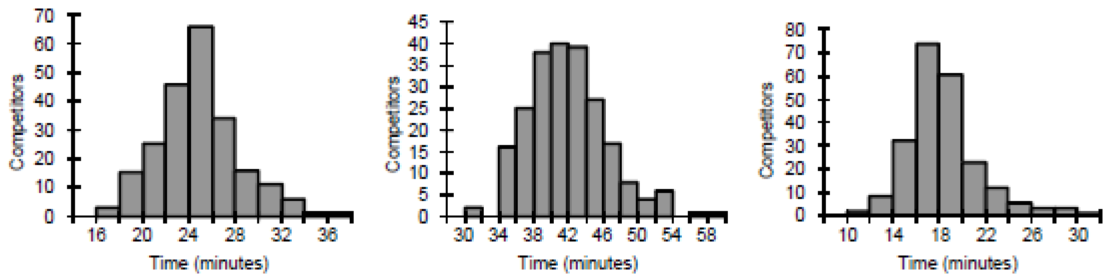
Question 8 **2014 Sample Paper 2 Q7**

The King of the Hill triathlon race in Kinsale consists of a 750 metre swim, followed by a 20 kilometre cycle, followed by a 5 kilometre run. The questions below are based on data from 224 athletes who completed this triathlon in 2010. Máire is analysing data from the race, using statistical software. She has a data file with each competitor’s time for each part of the race, along with various other details of the competitors. Máire gets the software to produce some summary statistics and it produces the following table. Three of the entries in the table have been removed and replaced with question marks (?).

	Swim	*Cycle*	*Run*
Mean	18.329	41.927	?
Median	17.9	41.306	?
Mode	#N/A	#N/A	#N/A
Standard Deviation	?	4.553	3.409
Sample Variance	10.017	20.729	11.622
Skewness	1.094	0.717	0.463
Range	19.226	27.282	20.87
Minimum	11.35	31.566	16.466
Maximum	30.576	58.847	37.336
Count	224	224	224

Máire produces histograms of the times for the three events. Here are the three histograms, without their titles.



(a)

i. Use the summary statistics in the table to decide which histogram corresponds to each event. Write the answers above the histograms.

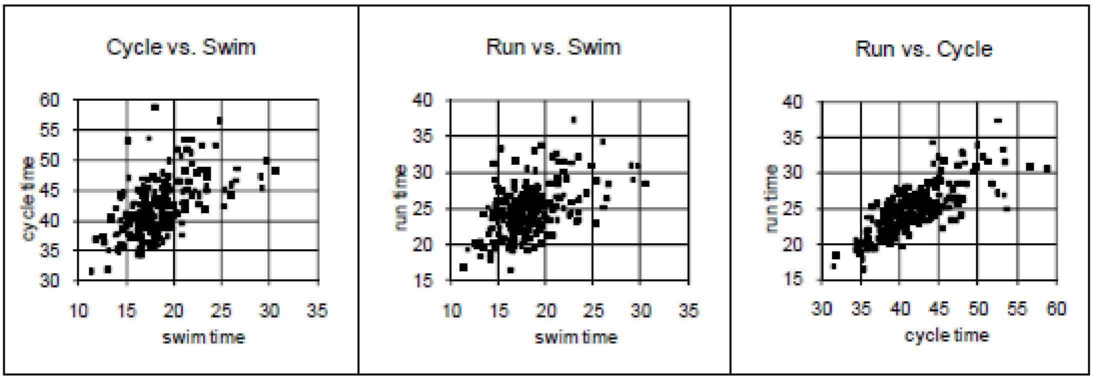
ii. The mean and the median time for the run are approximately equal. Estimate this value from the corresponding histogram.

mean ≈ median ≈ _______________

iii. Estimate from the relevant histogram the standard deviation of the times for the swim.
standard deviation ≈ _______________

iv. When calculating the summary statistics, the software failed to find a mode for the data sets. Why do you think this is?

Máire is interested in the relationships between the athletes' performance in the three different events. She produces the following three scatter diagrams



(b) Give a brief summary of the relationship between performance in the different events, based on the scatter diagrams.

(c) The best-fit line for run-time based on swim-time is y = 0.53*x* +15.2. The best-fit line for run-time based on cycle-time is y = 0.58*x* + 0.71. Brian did the swim in 17.6 minutes and the cycle in 35.7 minutes. Give your best estimate of Brian's time for the run, and justify your answer.

The mean finishing time for the overall event was 88.1 minutes and the standard deviation was 10.3 minutes.

(d) Based on an assumption that the distribution of overall finishing times is approximately normal, use the empirical rule to complete the following sentence:

"95% of the athletes took between ______ and ______ minutes to complete the race."

(e) Using normal distribution tables, estimate the number of athletes who completed the race in less than 100 minutes.

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Question 9 **2015 Sample Paper 2 Q2**

A survey of 100 shoppers, randomly selected from a large number of Saturday supermarket shoppers, showed that the mean shopping spend was €90.45. The standard deviation of this sample was €20.73.

(a) Find a 95% confidence interval for the mean amount spent in a supermarket on that Saturday. Explain what the confidence interval means.

(b) A supermarket has claimed that the mean amount spent by shoppers on a Saturday is €94. Based on the survey, test the supermarket's claim using a 5% level of significance.
Clearly state your null hypothesis, your alternative hypothesis, and your conclusion.

(c) Find the p-value of the test you performed in part (b) above and explain what this value represents in the context of the question.

Question 10 **2015 Sample Paper 2 Q2**

A machine produces packets of sweets and the packet says that the weight is 125g. In practice the actual weight, x, of a packet is random. Over a period when the machine was known to be working well the weights of a sample of 1,000 packets were measured:

Weight group $x \leq$	117	121	123	125	127	129	133	141
Frequency	0	67	138	274	327	102	45	47

For each group, the range of weights is previous limit < $x \leq$ group limit.

(a) Draw a histogram to illustrate this data.

(b) How would you construct a probability distribution for x from this data? Adjust your diagram in (a) accordingly, visually drawing a continuous curve.

(c) How would you use this to estimate the probability that a packet taken at random while the machine is working well would weigh between x_1 and x_2?

This sample of 1,000 weights is regarded as large enough for its mean and standard deviation to be taken as the mean and standard deviation of the x population when the machine is working well, and these have been calculated as $\mu_x = 125.38$ and $\sigma_x = 3.66$.

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A quality control process is being introduced. It consists of periodically taking a sample of 50 packets and finding their average weight $\overline{x}$. This process is carried out a few times a day every day.

(d) For the population of average weights $\overline{x}$ of samples of 50 packets,

i. what is the mean $\mu_{\overline{x}}$?

ii. what is the standard deviation $\sigma_{\overline{x}}$?

iii. what is the distribution of $\overline{x}$?

iv. superimpose a rough sketch of the distribution of $\overline{x}$ on a rough sketch of the distribution of x and comment on the comparison.

(e) A sample of 50 packets is taken and the value of $\overline{x}$ for this sample is 124.32. What is the probability that a deviation from the mean as large as this is observed even though the machine is working well?

(f) Under the quality control process, the machine is to be stopped for maintenance when a 50 packet sample is taken and the observed $\overline{x}$ deviates by more than D from 125.38. The management will accept a 0.5% probability that the machine is stopped for maintenance even when it is working well.
Calculate D.

Solutions: https://web.actuaries.ie/students/maths-tutorials-higher-level-leaving-certificate-20192020 or google 'actuaries maths tutorials'