



Probability Tutorial Additional Questions 2020/2021

Question 1 [Revision Question]

- a) How many different arrangements can be formed from the letters of the word CHEMISTRY?
- b) How many different ways can the letters be arranged if:
 - i. The first letter is a vowel and the last letter is Y
 - ii. The letters C and H must be together
 - iii. The letters C and H must not be together
- c) In how many ways can 6 numbers be chosen from 42?
- d) In a 20 team league, how many matches can be organised if:
 - i. Each team plays each other exactly once
 - ii. Each team plays each other home and away

Question 2 [Revision Question]

A bag contains 3 white beads, 4 blue beads and 3 black beads.

Two beads are chosen at random and not replaced.

- a) Find the probability that the beads chosen are:
 - i. Both black
 - ii. Black and blue in that order
 - iii. Black and blue in any order
- b) Find the probability that at least one of the beads is white.

Question 3

[2014 SEC Sample, Paper 2, Q1] (25 marks)

The random variable X has a discrete distribution. The probability that it takes a value other than 13, 14, 15 or 16 is negligible.

- a) Complete the probability distribution table below and hence calculate $E(X)$, the expected value of X .

x	13	14	15	16
$P(X=x)$	0.383	0.575		0.004

- b) If X is the age, in complete years, on 1 January 2013 of a student selected at random from among all second year students in Irish schools, explain what $E(X)$ represents.
- c) If ten students are selected at random from this population, find the probability that exactly six of them were 14 years old on 1 January 2013. Give your answer correct to three significant figures.

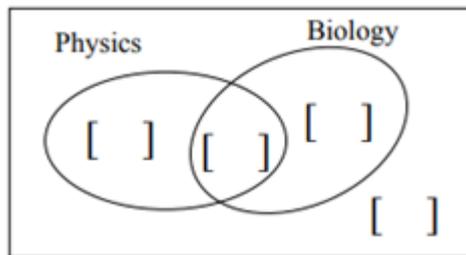


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Question 4

[2013 Paper 2, Q1] (25 marks)

- a) Explain each of the following terms:
- Sample space
 - Mutually exclusive events
 - Independent events
- b) In a class of 30 students, 20 study Physics, 6 study Biology and 4 study both Physics and Biology.
- Represent the information on the Venn Diagram



A student is selected at random from this class. The events E and F are:

- E: The student studies Physics
F: The student studies Biology

- By calculating probabilities, investigate if the events E and F are independent.

Question 5

[2017 Paper 2, Q1]

When Conor rings Ciara's house, the probability that Ciara answers is $\frac{1}{5}$.

- Conor rings Ciara's house once every day for 7 consecutive days. Find the probability that she will answer the phone on the 2nd, 4th and 6th days but not on the other days.
- Find the probability that she will answer the phone for the 4th time on the 7th day.
- Conor rings her house once every day for n days. Write, in terms of n , the probability that Ciara will answer the phone at least once.
- Find the minimum value of n for which the probability that Ciara will answer the phone at least once is greater than 99%.

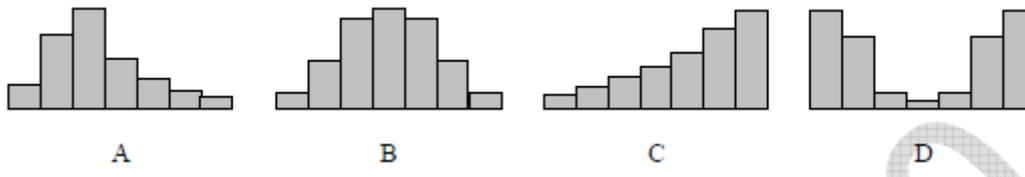


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Question 6

[2010 Sample Paper 2, Q2]

The shapes of the histograms of four different sets of data are shown below.



(a) Complete the table below, indicating whether the statement is correct (✓) or incorrect (✗) with respect to each data set.

	A	B	C	D
The data are skewed to the left				
The data are skewed to the right				
The mean is equal to the median				
The mean is greater than the median				
There is a single mode				

(b) Assume that the four histograms are drawn on the same scale.
State which of them has the largest standard deviation, and justify your answer.

Question 7

Using the standard normal tables

- (a) Find $P(Z \leq 2.43)$
- (b) Find $P(Z \geq 1.96)$
- (c) Find $P(Z \geq -0.5)$
- (d) Find $P(Z \leq -1.2)$



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Question 8 2014 Paper 2 Question 7

Table 1 below gives details of the number of males (M) and females (F) aged 15 years and over at work, unemployed, or not in the labour force for each year in the period 2004 to 2013.

Table 1										
Labour Force Statistics 2004 to 2013 - Persons aged 15 years and over (000's)										
Year	At work			Unemployed			Not in labour force			Total
	M	F	Total	M	F	Total	M	F	Total	
2004	1045.9	738.9	1784.8	79.6	31.6	111.2	457.1	854.2	1311.3	3207.3
2005	1087.3	779.7	1867	81.3	33.5	114.8	459.5	846.6	1306.1	3287.9
2006	1139.8	815.1	1954.9	80.6	38.1	118.7	457.6	844.9	1302.5	3376.1
2007	1184	865.6	2049.6	84.3	39.2	123.5	472.4	852.7	1325.1	3498.2
2008	1170.9	889.5	2060.4	106.3	41	147.3	494.8	872.5	1367.3	3575
2009	1039.8	863.5	1903.3	234	82.4	316.4	505.6	874.9	1380.5	3600.2
2010	985.1	843.5	1828.6	257.6	98.2	355.8	529.2	884.6	1413.8	3598.2
2011	970.2	843.2	1813.4	260.7	103.4	364.1	540.1	881.5	1421.6	3599.1
2012	949.6	823.8	1773.4	265.2	108	373.2	546.5	896.9	1443.4	3590
2013	974.4	829	1803.4	227.7	102.3	330	557.8	895	1452.8	3586.2

- (a) Suggest two categories of people, aged 15 years and over, who might not be in the labour force.
- (b) Find the median and the interquartile range of the total persons at work over the period.
- (c) The following data was obtained from Table 1. The percentages of persons aged 15 years and over at work, unemployed, or not in the labour force for the year 2006 are given below.

		At work	Unemployed	Not in the labour force
Persons aged 15 years and over	2006	57.9%	3.5%	38.6%
	2011			

Complete the table for the year 2011. Give your answers correct to one decimal place.

- (d) A census in 2006 showed that there were 864 449 persons in the population aged under 15 years of age. The corresponding number in the 2011 census was 979 590. Assuming that none of these persons are in the labour force, complete the table below to give the percentages of the total population at work, unemployed, or not in the labour force for the year 2011.

		At work	Unemployed	Not in the labour force
Total population	2006	46.1%	2.8%	51.1%



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	2011				
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A commentator states that “The changes reflected in the data from 2006 to 2011 make it more difficult to balance the Government’s income and expenditure.”

Do you agree with this statement? Give two reasons for your answer based on your calculations above.

Question 9 2015 Sample Paper Question 2

A survey done 10 years ago showed that 26% of the students of a large school lived within 2.5 km of the school. The principal of the school claims that the proportion of students living within 2.5km of the school is no longer 26%. From a random sample of 60 current students from the school it was found that 20% lived within 2.5 km.

- (a) Test the principal’s claim using a 5% level of significance.
Clearly state your null hypothesis, your alternative hypothesis and your conclusion.
- (b) Based on the sample data for the 60 students, find the 95% confidence interval for the proportion of students from that school who live within 2.5 km of the school.

Question 10 2011 Paper 2 Question 7

- (a) Some students are using a database of earthquakes to investigate the times between the occurrences of serious earthquakes around the world. They extract information about all of the earthquakes in the 20th century that caused at least 1000 deaths. There are 115 of these.

The students wonder whether there are patterns in the timing of these earthquakes, so they look at the number of days between each successive pair of these earthquakes.

They make the following table, showing the number of earthquakes for which the time interval from the previous earthquake is as shown.

Time in days from previous earthquake	0 – 100	100 – 200	200 – 300	300 – 400	400 – 500	500 – 600	600 – 700	700 – 800	800 – 1000	1000 – 1300
Number of earthquakes	31	24	12	14	8	7	5	6	5	3

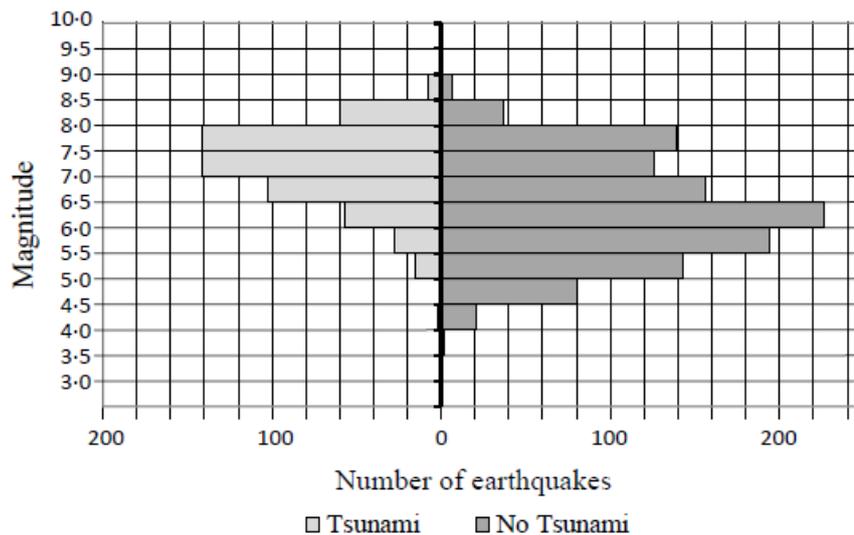
[Source: National geophysical data center, significant earthquake database: www.ngdc.noaa.gov]

- (i) Create a suitable graphical representation of the distribution.
- (ii) Describe the distribution. Your description should refer to the shape of the distribution and should include an estimate of the median.



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- (iii) The mean time between these earthquakes is 309 days and the standard deviation is 277 days. Suppose that such an earthquake has just occurred and that we want to find the probability that the time to the next one will be between 100 and 200 days. Explain why it would **not** be correct to use standard normal distribution tables (*z*-tables) to do this.
- (iv) Based on the information presented in this question so far, what is the best estimate for the probability described in part (iii) above? Explain your reasoning.
- (v) As stated at the beginning, the students chose to analyse earthquake timings by looking at the time intervals between the occurrences of a particular type of earthquake. Suggest a different way that they could have looked at the data in the database in order to try to find patterns in the timing of earthquakes.
- (b) The students heard a reporter saying that “strong earthquakes will cause large destructive ocean waves called tsunamis, while weaker ones will not”. They decide to check this. They draw two histograms back to back, one showing the magnitudes of the earthquakes that caused tsunamis, and the other showing the magnitudes of those that did not. They use all of the suitable data from the 20th century that were recorded in this particular database.



- (i) Comment on the reporter’s statement, using information from the diagram to support your answer, and suggest a more accurate statement.
- (ii) By taking suitable readings from the diagram, estimate the probability that an earthquake of magnitude between 6.5 and 7.0 will cause a tsunami.
- (iii) Consider the next six earthquakes of magnitude at least 7.5. Find an estimate for the probability that at least four of them will cause a tsunami, assuming that these six events are independent of each other.



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Question 11 2017 Paper 2 Question 2

An experiment measures the fuel consumption at various speeds for a particular model of car. The data collected are shown in Table 1 below.

Speed (km/hour)	40	48	56	64	88	96	112
Fuel consumption (km/litre)	21	16	18	16	13	11	9

- (a) Find the correlation coefficient of the data in Table 1, correct to 3 decimal places.
- (b) Plot the points from the table on a grid and draw the line of best fit (by eye).
- (c) The slope of the line of best fit is found to be -0.15 . What does this value represent in the context of the data?
- (d) Mary drove from Cork to Dublin at an average speed of 96 km/h.
Jane drove the same journey at an average speed of 112 km/h.
Each travelled 260 km and paid 132.9 cents per litre for the fuel.
Both used the model of car used to generate the data in Table 1.
- (i) Find how much longer it took Mary to complete the journey.
Give your answer correct to the nearest minute.
- (ii) Based on the data in Table 1 and their average speeds, find how much more Jane spent on fuel during the course of this journey.

Solutions: <https://web.actuaries.ie/students/maths-tutorials-higher-level-leaving-certificate-20192020> or google 'actuaries maths tutorials'