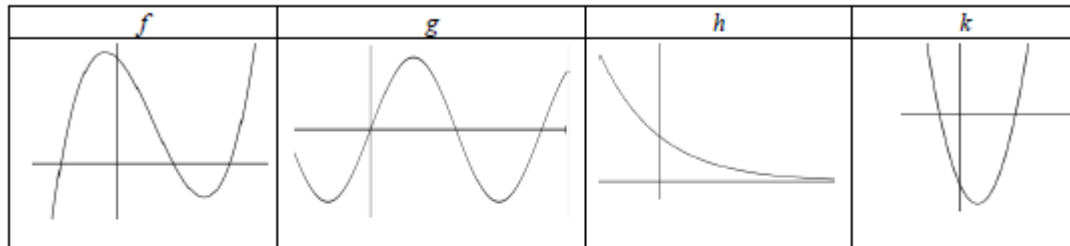




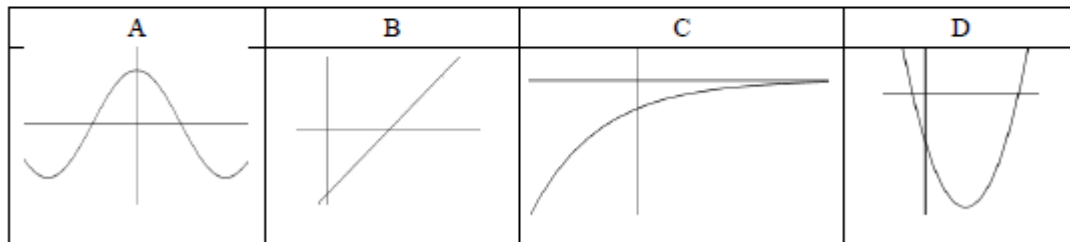
Question 1

[2013 Sample Paper 1, Q5] (25 marks)

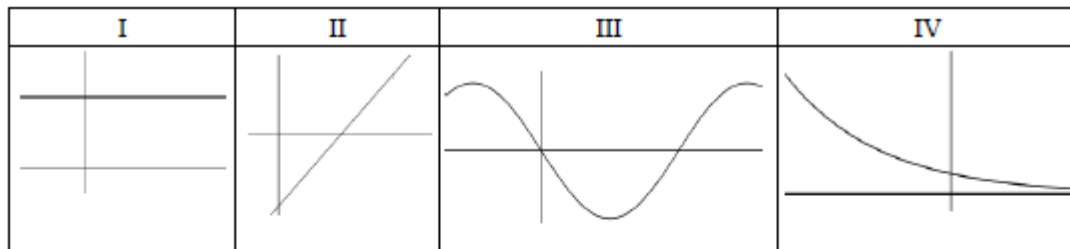
Each diagram below shows part of the graph of a function. Each is one of these: quadratic, cubic, trigonometric or exponential (not necessarily in that order).



Each diagram below shows part of the graph of the first derivative of one of the above functions (not necessarily in the same order).



Each diagram below shows part of the graph of the second derivative of one of the original functions (not necessarily in the same order).



(a) Complete the table below by matching the function to its first derivative and its second derivative.

Type of function	Function	First derivative	Second derivative
Quadratic	$k$	B	I
Cubic	$f$	D	II
Trigonometric	$g$	A	III
Exponential	$h$	C	IV

(b) For one row in the table, explain your choice of first derivative and second derivative.

A quadratic function differentiates to a line which differentiates to a constant.



Question 2

[2012 Paper 1, Q6] (25 marks)

(a) Let  $f(x) = e^{-\frac{1}{2}x^2}$ .

Show that the second derivative of  $f(x)$  with respect to  $x$  is  $f''(x) = (x^2 - 1)e^{-\frac{1}{2}x^2}$ .

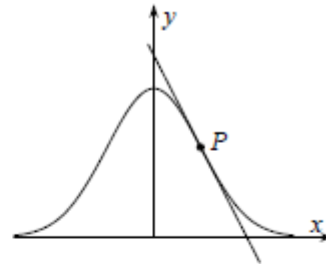
$$f'(x) = \frac{-x e^{-\frac{1}{2}x^2}}{1}$$

$$f''(x) = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$= -x \left( -x e^{-\frac{1}{2}x^2} \right) + e^{-\frac{1}{2}x^2} (-1)$$

$$= (x^2 - 1) e^{-\frac{1}{2}x^2}$$

- (b) The point  $P$  in the first quadrant is a point of inflexion of the curve  $y = e^{-\frac{1}{2}x^2}$ .  
Show that the tangent at  $P$  crosses the  $x$ -axis at  $(2, 0)$ .



Point of inflexion when  $f''(x) = 0$ .

$$(x^2 - 1)e^{-\frac{1}{2}x^2} = 0 \Rightarrow (x^2 - 1) = 0 \Rightarrow x = \pm 1.$$

First quadrant  $\Rightarrow x = 1 \Rightarrow y = f(1) = e^{-\frac{1}{2}}$ , and slope of tangent is  $f'(1) = -e^{-\frac{1}{2}}$

So the equation of the tangent is  $y - y_1 = m(x - x_1)$

$$y - e^{-\frac{1}{2}} = -e^{-\frac{1}{2}}(x - 1)$$

Crosses  $x$ -axis when  $y = 0 \Rightarrow -e^{-\frac{1}{2}} = -e^{-\frac{1}{2}}(x - 1)$

$$1 = x - 1$$

$$x = 2$$



Question 3

[2015 Paper 1, Q3] (25 marks)

Let  $f(x) = -x^2 + 12x - 27$ ,  $x \in \mathbb{R}$ .

(a) (i) Complete Table 1 below.

Table 1							
$x$	3	4	5	6	7	8	9
$f(x)$	0	5	8	9	8	5	0

(ii) Use Table 1 and the trapezoidal rule to find the approximate area of the region bounded by the graph of  $f$  and the  $x$ -axis.

$$\begin{aligned}
 A &= \frac{h}{2} [y_1 + y_n + 2(y_2 + y_3 + \dots + y_{n-1})] \\
 &= \frac{1}{2} [0 + 0 + 2(5 + 8 + 9 + 8 + 5)] \\
 &= 35 \text{ square units}
 \end{aligned}$$

(b) (i) Find  $\int_3^9 f(x) dx$ .

$$\begin{aligned}
 &\int_3^9 (-x^2 + 12x - 27) dx \\
 &= \left[ \frac{-x^3}{3} + \frac{12x^2}{2} - 27x \right]_3^9 \\
 &= (-243 + 486 - 243) - (-9 + 54 - 81) \\
 &= 36
 \end{aligned}$$

(ii) Use your answers above to find the percentage error in your approximation of the area, correct to one decimal place.

$$\frac{1}{36} \times 100 = 2.8\%$$



Question 4

[2014 Paper 1, Q5] (25 marks)

- (a) Find  $\int 5 \cos 3x \, dx$ .

$$\int 5 \cos 3x \, dx = \frac{5}{3} \sin 3x + c$$

- (b) The slope of the tangent to a curve  $y = f(x)$  at each point  $(x, y)$  is  $2x - 2$ .  
The curve cuts the  $x$ -axis at  $(-2, 0)$ .

- (i) Find the equation of  $f(x)$ .

$$\begin{aligned} \int dy &= \int (2x - 2) dx \\ \Rightarrow y &= x^2 - 2x + c \\ \text{At } x = -2, y = 0 &\Rightarrow 0 = 4 + 4 + c \Rightarrow c = -8 \\ \text{Hence, } y &= x^2 - 2x - 8 \end{aligned}$$

- (ii) Find the average value of  $f$  over the interval  $0 \leq x \leq 3, x \in \mathbb{R}$ .

$$\begin{aligned} \text{Average value: } \frac{1}{b-a} \int_a^b f(x) dx \\ \frac{1}{3-0} \int_0^3 (x^2 - 2x - 8) dx &= \frac{1}{3} \left[ \frac{x^3}{3} - x^2 - 8x \right]_0^3 \\ &= \frac{1}{3} \left[ \frac{27}{3} - 9 - 24 \right] = -8 \end{aligned}$$



Question 5

[2015 Paper 1, Q5] (25 marks)

- (a) Solve the equation  $x = \sqrt{x+6}$ ,  $x \in \mathbb{R}$

$$\begin{aligned}x &= \sqrt{x+6} \\ \Rightarrow x^2 &= x+6 \\ \Rightarrow x^2 - x - 6 &= 0 \\ \Rightarrow (x+2)(x-3) &= 0 \\ \Rightarrow x &= -2, \quad x = 3 \\ x = -2: \quad -2 &\neq \sqrt{-2+6} = \sqrt{4} = 2 \quad \times \\ x = 3: \quad 3 &= \sqrt{3+6} = \sqrt{9} = 3 \quad \checkmark\end{aligned}$$

- (b) Differentiate  $x - \sqrt{x+6}$  with respect to  $x$ .

$$\begin{aligned}f(x) &= x - \sqrt{x+6} = x - (x+6)^{\frac{1}{2}} \\ f'(x) &= 1 - \frac{1}{2}(x+6)^{-\frac{1}{2}} = 1 - \frac{1}{2\sqrt{x+6}}\end{aligned}$$

- (c) Find the co-ordinates of the turning point of the function  $y = x - \sqrt{x+6}$ ,  $x \geq -6$ .

$$\begin{aligned}f'(x) = 0 &\Rightarrow 1 - \frac{1}{2\sqrt{x+6}} = 0 \\ \Rightarrow 2\sqrt{x+6} &= 1 \\ \Rightarrow x+6 &= \frac{1}{4} \\ \Rightarrow x &= -5\frac{3}{4} \\ f(-5\frac{3}{4}) &= -5\frac{3}{4} - \sqrt{\frac{1}{4}} = -6\frac{1}{4} \\ (-5\frac{3}{4}, -6\frac{1}{4})\end{aligned}$$



Question 6

[2016 Paper 1, Q7] (40 marks)

Q7	Model Solution – 40 Marks	Marking Notes
(a) (i)	$v = \frac{4}{3}\pi r^3 \Rightarrow \frac{dv}{dr} = 4\pi r^2$ $\frac{dv}{dt} = 250 \text{ cm}^3/\text{s}$ $\frac{dr}{dt} = \frac{dr}{dv} \cdot \frac{dv}{dt} = \frac{1}{4\pi r^2} \cdot 250$ $\frac{dr}{dt} = \frac{250}{4\pi 400} = \frac{5}{32\pi} \text{ cm/s}$	<p>Scale 10C (0, 3, 7, 10)</p> <p><i>Low Partial Credit</i></p> <ul style="list-style-type: none"> <li>work towards <math>\frac{dv}{dr}</math> or <math>\frac{dv}{dt}</math> or <math>\frac{dr}{dt}</math></li> </ul> <p><i>High Partial Credit</i></p> <ul style="list-style-type: none"> <li>correct expression for <math>\frac{dr}{dt}</math></li> </ul>
(ii)	$a = 4\pi r^2 \Rightarrow \frac{da}{dr} = 8\pi r$ $\frac{da}{dt} = \frac{da}{dr} \cdot \frac{dr}{dt} = 8\pi r \cdot \frac{5}{32\pi}$ $= \frac{5(20)}{4}$ $= 25 \text{ cm}^2/\text{s}$	<p>Scale 10C (0, 3, 7, 10)</p> <p><i>Low Partial Credit</i></p> <ul style="list-style-type: none"> <li>work towards <math>\frac{da}{dr}</math> or <math>\frac{da}{dt}</math></li> </ul> <p><i>High Partial Credit</i></p> <ul style="list-style-type: none"> <li>correct expression for <math>\frac{da}{dt}</math></li> </ul>
(b) (i)	$-x^2 + 10x = 0$ $x(-x + 10) = 0$ $x = 0 \text{ or } x = 10$	<p>Scale 10C (0, 3, 7, 10)</p> <p><i>Low Partial Credit</i></p> <ul style="list-style-type: none"> <li>quadratic equation formed</li> <li>gets <math>x = 0</math> only</li> </ul> <p><i>High Partial Credit</i></p> <ul style="list-style-type: none"> <li>quadratic factorised</li> </ul> <p>Note: <math>f'(x) = 0 \Rightarrow 2x - 10 = 0 \Rightarrow x = 5</math> merits 0 marks</p>
(ii)	$\frac{1}{10-0} \int_0^{10} (-x^2 + 10x) dx$ $= \frac{1}{10} \left[ \frac{-x^3}{3} + 5x^2 \right]_0^{10}$ $= \frac{1}{10} \left[ \left( \frac{-1000}{3} + 500 \right) - 0 \right]$ $= \frac{-100}{3} + 50 = \frac{50}{3} \text{ m}$	<p>Scale 10C (0, 3, 7, 10)</p> <p><i>Low Partial Credit</i></p> <ul style="list-style-type: none"> <li>integration set up</li> </ul> <p><i>High Partial Credit</i></p> <ul style="list-style-type: none"> <li>correct integration with some substitution</li> </ul>



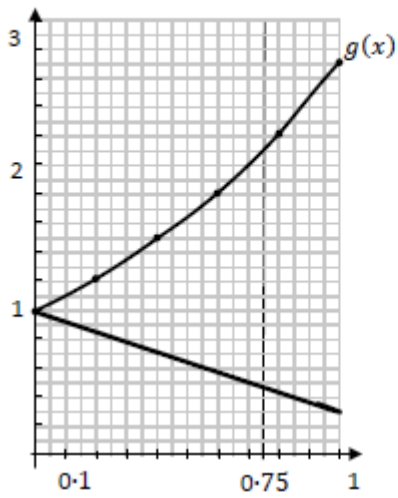
Question 7

[2017 Paper 1, Q6] (25 marks)

Q6	Model Solution – 25 Marks	Marking Notes																												
(a)	<p style="text-align: center;"> <math>g(x) = e^x \quad h(x) = e^{-x} = \frac{1}{e^x}</math> </p> <p><math>g(x) = e^x</math>:</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>x</td> <td>0</td> <td>0.2</td> <td>0.4</td> <td>0.6</td> <td>0.8</td> <td>1.0</td> </tr> <tr> <td>y</td> <td>1</td> <td>1.22</td> <td>1.49</td> <td>1.82</td> <td>2.23</td> <td>2.72</td> </tr> </table> <p><math>h(x) = \frac{1}{e^x}</math>:</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>x</td> <td>0</td> <td>0.2</td> <td>0.4</td> <td>0.6</td> <td>0.8</td> <td>1.0</td> </tr> <tr> <td>y</td> <td>1</td> <td>0.82</td> <td>0.67</td> <td>0.55</td> <td>0.45</td> <td>0.37</td> </tr> </table>	x	0	0.2	0.4	0.6	0.8	1.0	y	1	1.22	1.49	1.82	2.23	2.72	x	0	0.2	0.4	0.6	0.8	1.0	y	1	0.82	0.67	0.55	0.45	0.37	<p>Scale 15C (0, 5, 10, 15)</p> <p><i>Low Partial Credit:</i></p> <ul style="list-style-type: none"> <li>• one point correct</li> </ul> <p><i>High Partial Credit</i></p> <ul style="list-style-type: none"> <li>• Graph not in required domain</li> </ul>
x	0	0.2	0.4	0.6	0.8	1.0																								
y	1	1.22	1.49	1.82	2.23	2.72																								
x	0	0.2	0.4	0.6	0.8	1.0																								
y	1	0.82	0.67	0.55	0.45	0.37																								



(b)



$$\begin{aligned} A &= \int_0^{0.75} e^x dx - \int_0^{0.75} e^{-x} dx \\ &= \int_0^{0.75} (e^x - e^{-x}) dx \\ &= e^x + e^{-x} \\ &= e^{0.75} + e^{-0.75} - [e^0 + e^0] \\ &= 0.5894 \end{aligned}$$

Scale 10C (0, 5, 8, 10)

*Low Partial Credit:*

- Formulates integration for area under one curve with limits

*High Partial Credit*

- integrates twice for correct area under both curves

**Note:** Trapezoidal rule must have at least 5 divisions **AND** fully correct work gets Low Partial Credit