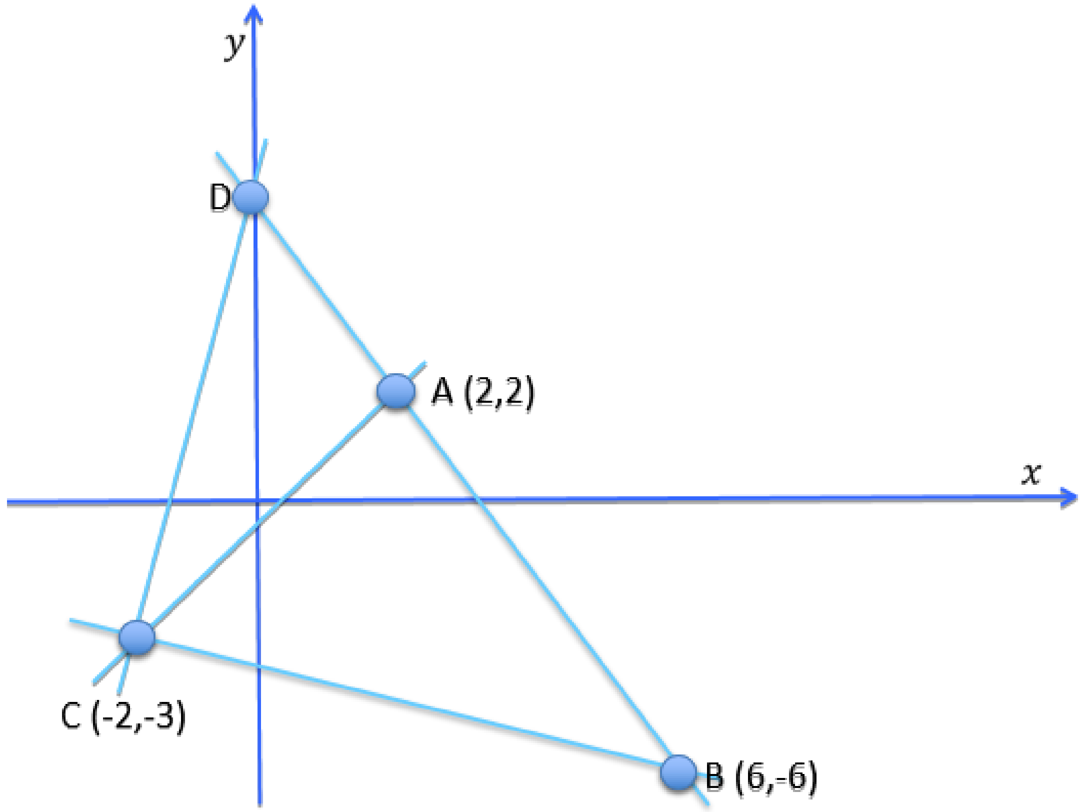
Geometry 2 Tutorial – Solutions

Q1.



a) Find the slope using slope formula

⇒ $m = \frac{(y2-y1)}{(x2-x1)} = \frac{(-6-2)}{(6-2)} = \frac{-8}{4} = -2$

⇒ Slope = -2, A = (2,2)

⇒ Equation of AB: y - 2 = -2 * (x - 2)

⇒ y - 2 = -2x +4

⇒ 2x + y = 6

b) Line intersects the y-axis means that x=0 at this point

⇒ Substitute x=0 into Equation of AB: 2x + y = 6

⇒ y = 6

⇒ D = (0,6)

c) *Formula in Tables Page 19*

C = (-2, -3) and AB = 2x + y = 6 or 2x + y – 6 = 0

$$\Rightarrow \boldsymbol{Distance} = \frac{|ax1+by1+c|}{\sqrt{(a^2+b^2)}} = \frac{|2(-2)+1(-3)-6|}{\sqrt{(2^2+1^2)}} = \frac{|-13|}{\sqrt{5}} = \frac{13}{\sqrt{5}}$$

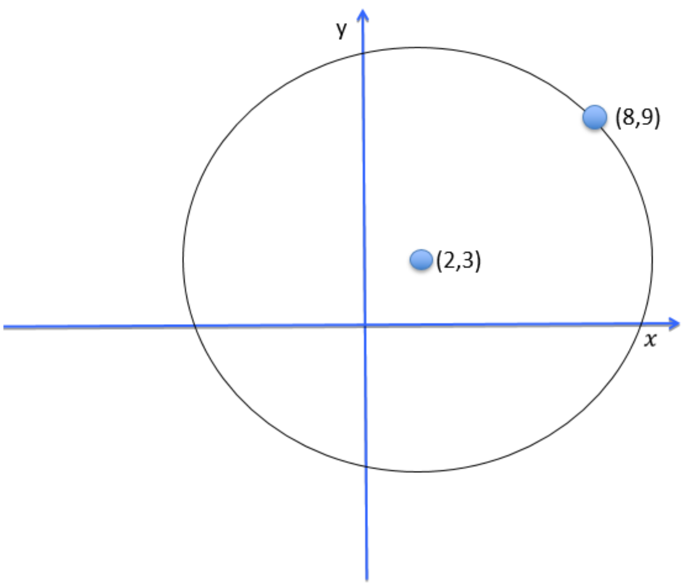
d) *Area of Triangle is ½ x base x perpendicular height*

- $\Rightarrow$ Perpendicular height is $\frac{13}{\sqrt{5}}$ from last question
- $\Rightarrow$ Distance between A(2,2) and D(0,6) is the base
- $\Rightarrow \sqrt{(0-2)^2 + (6-2)^2}$
- $\Rightarrow \sqrt{4+16} = \sqrt{20}$
- $\Rightarrow$ Area = ½ * $\sqrt{20}$* $\frac{13}{\sqrt{5}}$ = 13 units squared

Note: To use formula for area of triangle on page 18 it is necessary to have one apex at the origin.

Q2.

a)



b) Distance between (8,9) and (2,3) is radius

$\Rightarrow \sqrt{(\mathbf{2-8})^2 + (\mathbf{3-9})^2}$

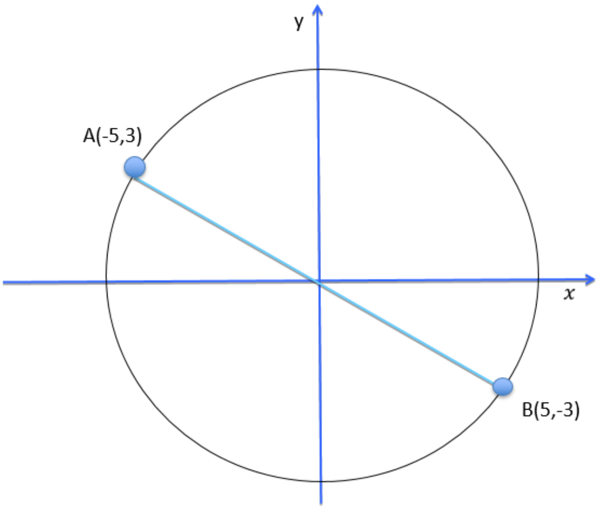
$\Rightarrow \sqrt{(\mathbf{-6})^2 + (\mathbf{-6})^2}$ = $\sqrt{\mathbf{36+36}}$=$\sqrt{\mathbf{72}}$

c) Equation of circle with centre (h,k) and radius r is: $(x-h)^2 + (y-k)^2 = r^2$

$\Rightarrow (x-2)^2 + (y-3)^2 = \sqrt{72}^2$=*72*

Q3.

a)



b) Midpoint of A(-5,3) and B(5,-3) is the centre of the circle

$\Rightarrow$ Midpoint is (0,0)

c) Distance between (-5,3) and (0,0) is radius

$\Rightarrow \sqrt{(-5-0)^2 + (3-0)^2}$

$\Rightarrow \sqrt{5^2 + 3^2}$

$\Rightarrow \sqrt{34}$

d) $x^2 + y^2 = r^2$

$\Rightarrow$ $x^2 + y^2 = (\sqrt{34})^2$

$\Rightarrow$ $x^2 + y^2 = 34$

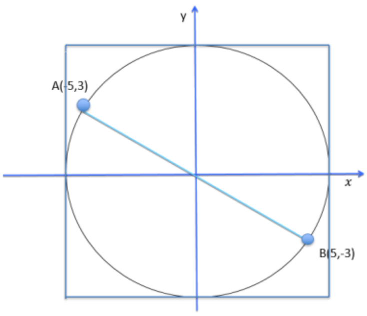
e) *Area* = πr^2

$\Rightarrow$ $\pi\sqrt{34}^2$

$\Rightarrow$ 34 π

$\Rightarrow$ 106.81 units squared

f)



Side of square is equal to the diameter, area of square is diameter ^{2}

$\Rightarrow$ *Diameter = 2r =* $2\sqrt{34}$

$\Rightarrow$ Area of square = $(2\sqrt{34})^2$

$\Rightarrow$ *136 units squared*

Q4.

a) *Solution circle c1*

$x^2 + y^2 + 2gx + 2fy + c = 0$

$\Rightarrow$ $x^2 + y^2 - 6x - 10y + 29 = 0$ *(Formulae p. 19)*

$\Rightarrow$ *2g = -6*

$\Rightarrow$ *-g = 3*

$\Rightarrow$ *2f = -10*

$\Rightarrow$ *-f = 5*

$\Rightarrow$ centre (3, 5)

$\Rightarrow$ Radius formula (p. 19)

$\Rightarrow$ $\sqrt{(g^2 + f^2 - c)}$, c = 29

$\Rightarrow$ $\sqrt{3^2 + 5^2 - 29}$

$\Rightarrow$ $\sqrt{9 + 25 - 29}$

$\Rightarrow$ $\sqrt{5}$

Solution circle c2

$x^2 + y^2 - 2x - 2y - 43 = 0$

$\Rightarrow$ 2g = -2

$\Rightarrow$ -g = 1

$\Rightarrow$ 2f = -2

$\Rightarrow$ -f = 1

$\Rightarrow$ centre (1,1)

$\Rightarrow$ Radius formula

$\Rightarrow$ $\sqrt{1^2 + 1^2 + 43}$

$\Rightarrow$ $\sqrt{45}$

$\Rightarrow$ $\sqrt{9 * 5}$

$\Rightarrow$ $3\sqrt{5}$

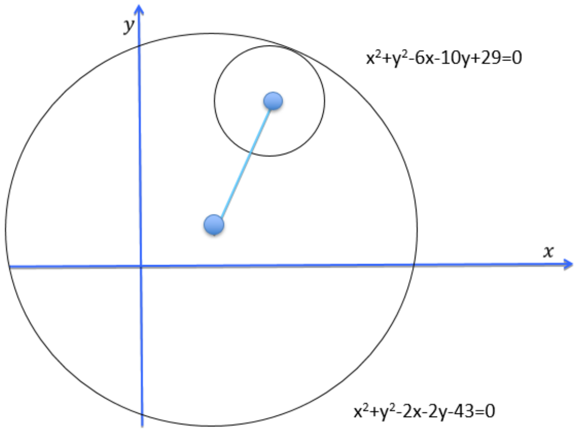
b) Distance between centres:

$\Rightarrow$ $\sqrt{(3-1)^2 + (5-1)^2}$

$\Rightarrow$ $\sqrt{20}$

$\Rightarrow$ $2\sqrt{5}$

$\Rightarrow$ The distance between the centres is the difference of the radii => circles touch (internally).



c) *Substitute (4,7) into c_1 and c_2*

$\Rightarrow$ $4^2 + 7^2 - 6(4) - 10(7) + 29 = 0$

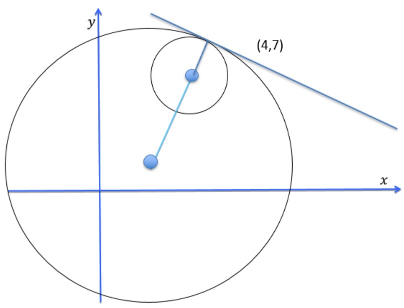
$\Rightarrow$ *(4, 7) is point on c_1*

$\Rightarrow$ $4^2 + 7^2 - 2(4) - 2(7) - 43 = 0$

$\Rightarrow$ *(4, 7) is point on c_2*

$\Rightarrow$ *Circles touch internally at (4,7)*

d)



Slope from (3, 5) to (4, 7) is: $\frac{7-5}{4-3} = 2$

$\Rightarrow$ *Slope of tangent =* $-\frac{1}{2}$ *(as perpendicular)*

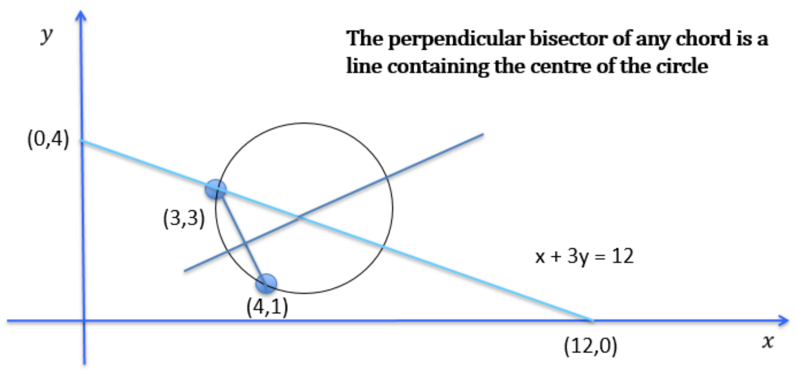
$\Rightarrow$ *Equation of tangent with slope* $-\frac{1}{2}$ *and point(4,7)*

$\Rightarrow$ $y - 7 = -\frac{1}{2}(x - 4)$

$\Rightarrow$ $2y - 14 = -x + 4$

$\Rightarrow$ $x + 2y - 18 = 0$

Q5.



The perpendicular bisector of any chord is a line containing the centre of the circle

$\Rightarrow$ End points of chord: (3,3) and (4,1)

$\Rightarrow$ Midpoint of Chord: (3.5, 2)

$\Rightarrow$ slope of chord: (1-3)/(4-3) = -2

$\Rightarrow$ slope of perpendicular of chord: $\frac{1}{2}$

Eqn of perpendicular of chord: slope = $\frac{1}{2}$, point (3.5,2)

$\Rightarrow$ $y - 2 = \frac{1}{2}(x - 3.5)$

$\Rightarrow$ $2y - 4 = x - 3.5$

$\Rightarrow$ $x - 2y = -0.5$

$\Rightarrow$ $\underline{x + 3y = 12}$ point of intersection is centre of circle

- $\Rightarrow -5y = -12.5$ *subtracting*
- $\Rightarrow y = 2.5;\ x = 4.5$
- $\Rightarrow$ *Centre is (4.5,2.5)*

Now, need to find the radius

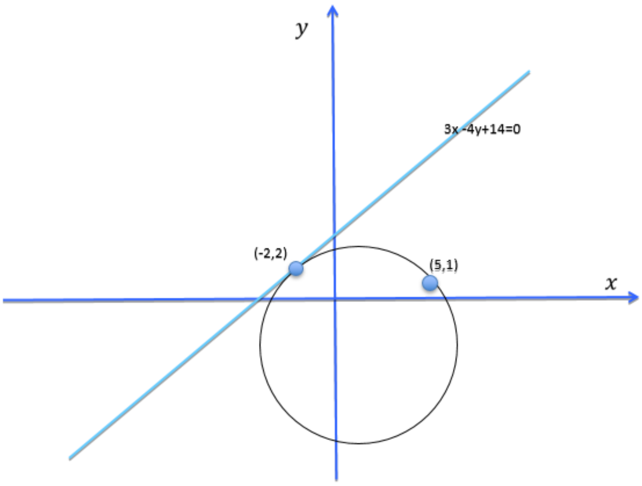
- $\Rightarrow$ Distance between (4.5,2.5) and (3,3)
- $\Rightarrow r = \sqrt{(3-4.5)^2 + (3-2.5)^2}$
- $\Rightarrow r = \sqrt{2.5}$

Equation of the circle:

- $\Rightarrow (x - 4.5)^2 + (y - 2.5)^2 = 2.5$
- $\Rightarrow x^2 + y^2 - 9x - 5y + 24 = 0$

Q6.

a)



b) Need to find the centre of the circle to find the equation of the circle

Know: The radius is perpendicular to the tangent at the point of intersection

Know: The perpendicular bisector of any chord is a line containing the centre of the circle

⇒ The point of intersection of these two lines is the centre of the circle

Tangent: 3x -4y+14=0

⇒ Slope of tangent = ¾

⇒ slope of perpendicular = -(4/3)

Equation of perpendicular: slope -4/3, point (-2,2)

⇒ $y - 2 = -(\frac{4}{3})(x - (-2))$

⇒ $3y - 6 = -4x - 8$

⇒ $4x + 3y = -2$

To find perpendicular bisector of chord:

⇒ Midpoint (-2,2) and (5,1): (1.5, 1.5)

⇒ Slope of (-2,2) and (5,1): (1-2)/(5-(-2))

⇒ Slope: -(1/7)

⇒ Slope of perpendicular = 7

Equation of perpendicular: slope 7, point (1.5,1.5)

⇒ $y - 1.5 = 7(x - 1.5)$

⇒ $7x - y = 9$

Simultaneous equations

⇒ $7x - y = 9$

⇒ $4x + 3y = -2$

⇒ $\underline{21x - 3y = 27}$

⇒ Adding: $25x = 25$;

⇒ $x = 1$ and $y = -2$

⇒ centre $(1, \ -2)$

⇒ So, centre (1,-2)

Radius is distance between (1,-2) and (5,1)

$\Rightarrow$ $r = \sqrt{(5-1)^2 + (1-(-2))^2}$

$\Rightarrow$ $r = \sqrt{16+9}$

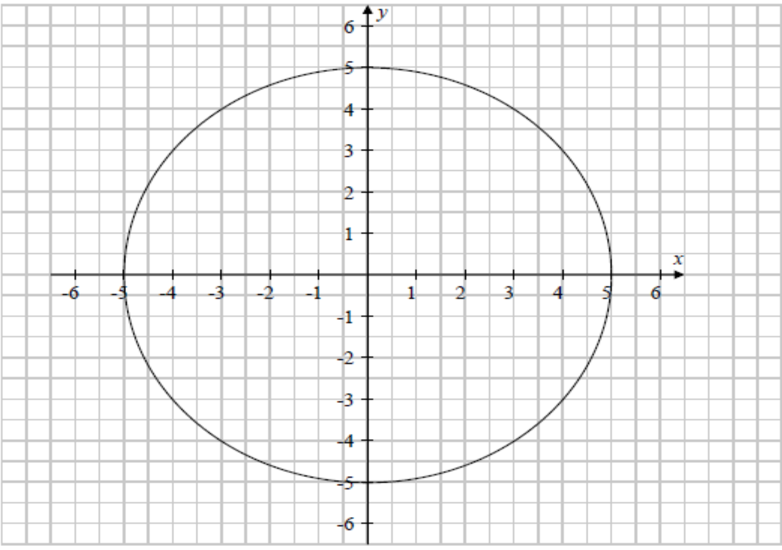
$\Rightarrow$ r = 5

$\Rightarrow$ Equation of circle: $(x-1)^2 + (y-(-2))^2 = 5^2$

$\Rightarrow$ $x^2 + y^2 - 2x + 4y - 20 = 0$

Q7.

a)



(b) Verify, using algebra, that $A(-4, 3)$ is on c.

$x^2 + y^2 = 25$

$(-4)^2 + 3^2 = 16 + 9 = 25 = \text{RHS}$

or

Centre of c: $O\,(0, 0)$

$|OA| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-4-0)^2 + (3-0)^2}$

$= \sqrt{25} = 5 =$ radius of c

(c) Find the equation of the circle with centre $(-4, 3)$ that passes through the point $(3, 4)$.

$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(3+4)^2 + (4-3)^2} = \sqrt{49+1} = \sqrt{50}$

$(x-h)^2 + (y-k)^2 = r^2 \Rightarrow \quad (x+4)^2 + (y-3)^2 = \left(\sqrt{50}\right)^2 = 50$

Q8.

a)

$\sqrt{g^2 + f^2 - c} = \sqrt{1+1+7} = 3$

(a) Complete the following table:

Circle	Centre	Radius	Equation
c_1	$(-3, -2)$	2	$(x+3)^2 + (y+2)^2 = 4$ **OR** $x^2 + y^2 + 6x + 4y + 9 = 0$
c_2	$(1, 1)$	3	$x^2 + y^2 - 2x - 2y - 7 = 0$

(b) **(i)** Find the co-ordinates of the point of contact of c_1 and c_2.

Divide line segment joining $(-3, -2)$ and $(1, 1)$ in ratio 2 : 3

$$\left(\frac{2(1)+3(-3)}{2+3}, \frac{2(1)+3(-2)}{2+3}\right) = \left(-\frac{7}{5}, -\frac{4}{5}\right)$$

OR

Slope line of centres $= \frac{3}{4}$.

Equation line of centres: $y - 1 = \frac{3}{4}(x-1) \Rightarrow 3x - 4y + 1 = 0$

$c_1 - c_2 = 4x + 3y + 8 = 0$

$4x + 3y + 8 = 0 \cap 3x - 4y + 1 = 0 \Rightarrow x = -\frac{7}{5}, y = -\frac{4}{5}$

(ii) Hence, or otherwise, find the equation of the tangent, t, common to c_1 and c_2.

Slope of line of centres: $\frac{1+2}{1+3}=\frac{3}{4}$

Slope of tangent: $m=-\frac{4}{3}$

Equation of tangent: $y+\frac{4}{5}=-\frac{4}{3}\left(x+\frac{7}{5}\right)$

$\Rightarrow\ 3y+\frac{12}{5}=-4x-\frac{28}{5}$

$\Rightarrow\ 4x+3y+8=0$

OR

$c_1-c_2=x^2+y^2+6x+4y+9-(x^2+y^2-2x-2y-7)=0$

$\Rightarrow 6x+4y+9-(-2x-2y-7)=0$

$\Rightarrow 8x+6y+16=0 \Rightarrow 4x+3y+8=0$

OR

$xx_1+yy_1+g(x+x_1)+f(y+y_1)+c=0$

$x(-\frac{7}{5})+y(-\frac{4}{5})+3(x+(-\frac{7}{5}))+2(y+(-\frac{4}{5})+9=0$

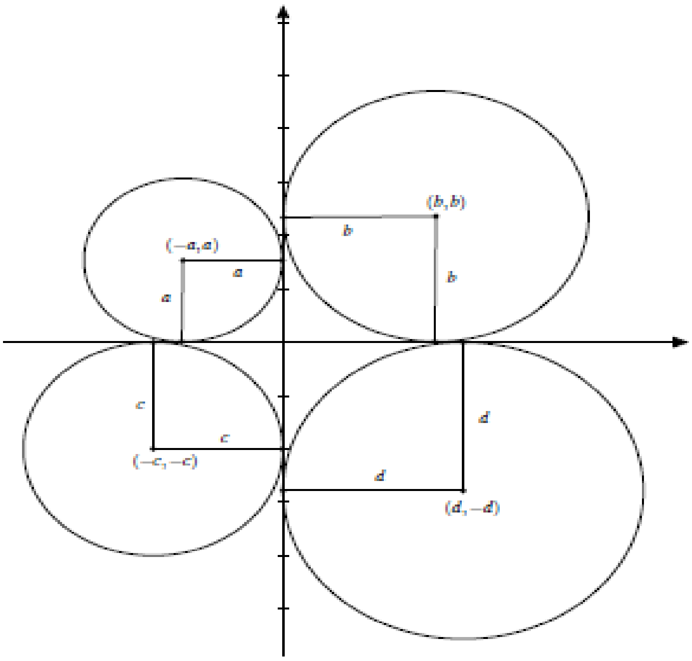
$\Rightarrow 4x+3y+8=0$

Q9.

a)

The centre of a circle lies on the line $x+2y-6=0$. The x-axis and the y-axis are tangents to the circle. There are two circles that satisfy these conditions. Find their equations.

Consider the diagram below:



Note that this diagram is not meant to represent the solution. It is just meant to illustrate the possibilities for circles that are tangent to both axes. In the diagram, a, b, c and d are the radii of the circles. We can see from this diagram that if (x,y) is the centre of a circle that has both the x-axis and the y-axis as tangents, then either

- Case 1: $y=x$
- Case 2: $y=-x$

In either case the radius is either x or $-x$. Since the radius is positive, we can say that the radius is $|x|$ in either case...

Case 1: $y = x$. We are also told that $x + 2y - 6 = 0$. Subsituting x for y in the latter equation gives

$$x + 2x - 6 = 0 \Leftrightarrow 3x - 6 = 0 \Leftrightarrow x = 2.$$

Now $y = x$ so $y = 2$. Therefore the centre of the circle has co-ordinates $(2, 2)$ and the radius is 2. Therefore in this case the circle has equation

$$(x-2)^2 + (y-2)^2 = 4.$$

Case 2: $y = -x$. As before we use this to substitute $-x$ for y in the equation $x + 2y - 6 = 0$. This gives

$$x + 2(-x) - 6 = 0 \Leftrightarrow -x - 6 = 0 \Leftrightarrow x = -6.$$

It follows that $y = -(-6) = 6$. So in this case the centre has co-ordinates $(-6, 6)$ and the radius is 6. So this circle has equation

$$(x+6)^2 + (y-6)^2 = 36.$$