



Warm Up Questions:

Question 1.

Let $f(x) = -x^2 + 12x - 27$, $x \in \mathbb{R}$.

(a) (i) Complete Table 1 below.

Table 1							
x	3	4	5	6	7	8	9
$f(x)$	0	5	8	9	8	5	0

Question 2. Solve for x : $\frac{x+7}{3} + \frac{2}{x} = 4$

Multiply across:

$$\Rightarrow \frac{(x+7)(x)(3)}{3} + \frac{2(x)(3)}{x} = 4(x)(3)$$

$$\Rightarrow x(x+7) + 2(3) = 12x$$

$$\Rightarrow x^2 + 7x + 6 = 12x$$

$$\Rightarrow x^2 + 7x - 12x + 6 = 0$$

$$\Rightarrow x^2 - 5x + 6 = 0$$

$$\Rightarrow (x - 3)(x - 2)$$

$$\Rightarrow x = 3, x = 2$$

Question 3. Express $\sqrt{48} - \sqrt{12} + \sqrt{27}$ in the form $a\sqrt{b}$

$$\Rightarrow \sqrt{16 \times 3} - \sqrt{4 \times 3} + \sqrt{9 \times 3}$$

$$\Rightarrow \sqrt{16}\sqrt{3} - \sqrt{4}\sqrt{3} + \sqrt{9}\sqrt{3}$$

$$\Rightarrow 4\sqrt{3} - 2\sqrt{3} + 3\sqrt{3}$$

$$\Rightarrow 5\sqrt{3}$$

Question 4. Simplify:

$$(b+1)^3 - (b-1)^3$$

$$(b+1)^3 = (b+1)(b+1)^2$$

$$= (b+1)(b^2 + 2b + 1)$$

$$= b(b^2 + 2b + 1) + 1(b^2 + 2b + 1)$$

$$= b^3 + 2b^2 + b + b^2 + 2b + 1$$

$$= b^3 + 3b^2 + 3b + 1 \quad \dots(1)$$

$$(b-1)^3 = (b-1)(b-1)^2$$

$$= (b-1)(b^2 - 2b + 1)$$



$$= b^3 - 3b^2 + 3b - 1 \quad \dots(2)$$

$$(b + 1)^3 - (b - 1)^3 = (1) - (2)$$

$$= b^3 + 3b^2 + 3b + 1 - (b^3 - 3b^2 + 3b - 1)$$

$$= 6b^2 + 2$$

Side note: It is useful to remember $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

Which you can use to check:

$$(b + 1)^3 - (b - 1)^3 = b^3 + 3b^2 + 3b + 1 - (b^3 - 3b^2 + 3b - 1)$$

$$= 6b^2 + 2$$

-b Formula

Question 5.

$$10x^2 + 6x - 52 = 0$$

$$a = 10 \quad b = 6 \quad c = -52$$

$$\Rightarrow \frac{-6 \pm \sqrt{6^2 - 4(10)(-52)}}{2(10)}$$

$$\Rightarrow \frac{-6 \pm \sqrt{36 - (-2080)}}{20}$$

$$\Rightarrow \frac{-6 \pm \sqrt{2116}}{20}$$

$$\Rightarrow \frac{-6 \pm 46}{20}$$

$$\Rightarrow \frac{-6 + 46}{20} = \frac{40}{20} = 2$$

$$\Rightarrow \frac{-6 - 46}{20} = \frac{-52}{20} = \frac{-13}{5}$$

$$X = 2 \text{ or } X = \frac{-13}{5}$$

Question 6. 2011 Paper 1 Q1

(c) Solve the equation $x^2 - 2\sqrt{3}x - 9 = 0$, giving your answers in the form $a\sqrt{3}$, where $a \in \mathbb{Q}$.

$$x = \frac{2\sqrt{3} \pm \sqrt{(-2\sqrt{3})^2 - 4(1)(-9)}}{2(1)} = \frac{2\sqrt{3} \pm \sqrt{48}}{2}$$

$$= \frac{2\sqrt{3} \pm 4\sqrt{3}}{2}$$

$$= \sqrt{3} \pm 2\sqrt{3}$$

$$x = -\sqrt{3} \text{ or } x = 3\sqrt{3}$$



Question 7. 2015 Paper 1 Q2 (25 marks)

Solve the equation $x^3 - 3x^2 - 9x + 11 = 0$.

Write any irrational solution in the form $a + b\sqrt{c}$, where $a, b, c \in \mathbb{Z}$.

Solution:

$$f(x) = x^3 - 3x^2 - 9x + 11$$

$$f(1) = 1^3 - 3(1)^2 - 9 + 11 = 0$$

$\Rightarrow x = 1$ is a solution.

$(x - 1)$ is a factor

$$\begin{array}{r} x-1 \overline{) \begin{array}{r} x^3 - 3x^2 - 9x + 11 \\ x^3 - x^2 \\ \hline -2x^2 - 9x + 11 \\ -2x^2 + 2x \\ \hline -11x + 11 \\ -11x + 11 \\ \hline 0 \end{array}} \end{array}$$

Hence, other factor is $x^2 - 2x - 11$

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-11)}}{2(1)} = \frac{2 \pm \sqrt{48}}{2} = \frac{2 \pm 4\sqrt{3}}{2} = 1 \pm 2\sqrt{3}$$

Solutions: $\{1, 1 + 2\sqrt{3}, 1 - 2\sqrt{3}\}$



Inequalities

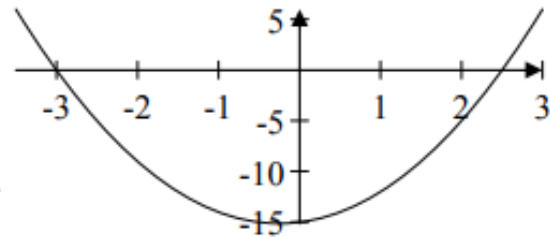
Question 8. 2013 Paper 1 Q2

(a) Find the set of all real values of x for which $2x^2 + x - 15 \geq 0$.

$$2x^2 + x - 15 = 0$$

$$\Rightarrow (2x - 5)(x + 3) = 0 \Rightarrow x = 2\frac{1}{2} \text{ or } x = -3$$

$$2x^2 + x - 15 \geq 0 \text{ for } \{x \mid x \leq -3\} \cup \{x \mid x \geq 2\frac{1}{2}\}$$



OR

$$f(x) = 2x^2 + x - 15 = (2x - 5)(x + 3)$$

$$(2x - 5)(x + 3) = 0$$

$$\Rightarrow x = \frac{5}{2} \text{ or } x = -3$$

$$(i): x \geq -3 \text{ and } x \geq \frac{5}{2} \Rightarrow x \geq \frac{5}{2}$$

$$(ii): x \leq -3 \text{ and } x \leq \frac{5}{2} \Rightarrow x \leq -3$$

$$\text{Solution Set: } \{x \mid x \leq -3\} \cup \{x \mid x \geq \frac{5}{2}\}$$

Question 9. Solve the following inequality and graph the solution, $x \in \mathbb{R}$:

$$|3x+4| \leq |x+2|$$

$$\text{Step 1: Solve } |3x+4| = |x+2|$$

$$(|3x + 4|)^2 = (|x + 2|)^2$$

$$\Rightarrow 9x^2 + 24x + 16 = x^2 + 4x + 4$$

$$\Rightarrow 8x^2 + 20x + 12 = 0$$

$$\Rightarrow 2x^2 + 5x + 3 = 0 \quad (\text{now your equation is in the form } ax^2 + bx + c)$$

$$\Rightarrow (2x + 3)(x + 1) = 0$$

$$\Rightarrow (2x + 3) = 0 \rightarrow x = -3/2$$

$$\Rightarrow (x + 1) = 0 \rightarrow x = -1$$

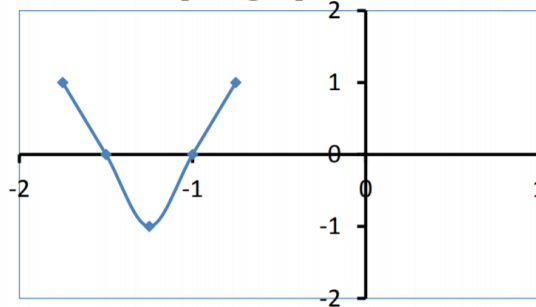
$$\Rightarrow \frac{-3}{2} \leq x \leq -1$$



Step 2:

$$a = 2$$

Since $a > 0$, use a U shaped graph

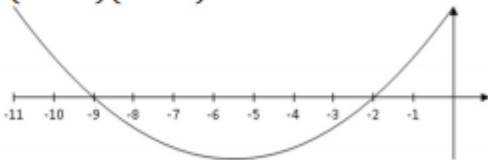


Question 10. 2018 Paper 1 Q1

(b) Solve the inequality $\frac{2x-3}{x+2} \geq 3$, where $x \in \mathbb{R}$ and $x \neq -2$.

$$\frac{2x-3}{x+2} \geq 3 \quad \times (x+2)^2 \quad \leftarrow \text{We multiply across by } (x+2)^2 \text{ as it is always non-negative}$$

$$\begin{aligned} (2x-3)(x+2) &\geq 3(x+2)^2 \\ 2x^2 + x - 6 &\geq 3x^2 + 12x + 12 \\ x^2 + 11x + 18 &\leq 0 \\ (x+2)(x+9) &\leq 0 \end{aligned}$$



$$-9 \leq x < -2$$

Question 11. 2012 Paper 1 Q1

(b) Find the set of all real values of x for which $\frac{2x-5}{x-3} \leq \frac{5}{2}$.

Multiply across by $2(x-3)^2$, which is non-negative:

$$\begin{aligned} 2(x-3)(2x-5) &\leq 5(x-3)^2 \\ 4x^2 - 22x + 30 &\leq 5x^2 - 30x + 45 \\ 0 &\leq x^2 - 8x + 15 \\ 0 &\leq (x-5)(x-3) \\ x &\geq 5 \text{ or } x < 3. \end{aligned}$$

or



$$\frac{2x-5}{x-3} - \frac{5}{2} \leq 0$$

$$\frac{2(2x-5) - 5(x-3)}{2(x-3)} \leq 0$$

$$\frac{-x+5}{2(x-3)} \leq 0$$

$$x \geq 5 \text{ or } x < 3.$$

	$x < 3$	$3 < x < 5$	$x > 5$
$-x+5$	+	+	-
$x-3$	-	+	+
$\frac{-x+5}{2(x-3)}$	-	+	-

Simultaneous Equations

Question 12.

Solve the simultaneous equations:

$$a^2 - ab + b^2 = 3$$

$$a + 2b + 1 = 0$$

$$a = -2b - 1$$

$$(-2b - 1)^2 + (2b + 1)b + b^2 = 3$$

$$7b^2 + 5b - 2 = 0$$

$$(7b - 2)(b + 1) = 0$$

$$b = \frac{2}{7} \text{ or } b = -1$$

$$a = \frac{-11}{7} \text{ or } a = 1$$

Solution: $\{b = \frac{2}{7} \text{ and } a = \frac{-11}{7}\}$ or $\{b = -1 \text{ and } a = 1\}$.

Question 13.

(a) Solve the simultaneous equations.

$$2x + 3y - z = -4$$

$$3x + 2y + 2z = 14$$

$$x - 3z = -13$$



Solution:

$$\begin{array}{rcl}
 \text{(i)} & 2x + 3y - z = -4 & \times (2) \\
 \text{(ii)} & 3x + 2y + 2z = 14 & \times (-3) \\
 & 4x + 6y - 2z = -8 \\
 & \underline{-9x - 6y - 6z = -42} \\
 & -5x - 8z = -50 \\
 \text{(iii)} & \underline{x - 3z = -13} & \times (5) \\
 & -5x - 8z = -50 \\
 & \underline{5x - 15z = -65} \\
 & -23z = -115 \\
 & z = 5 \\
 & \Rightarrow x = 2 \\
 & \Rightarrow y = -1 & \{2, -1, 5\}
 \end{array}$$

Logs

Question 14. Solve $\log_x 8 = 3$

$$x^3 = 8$$

$$x = 2$$

Question 15. Solve $32^{x-1} = 28$ for x and give your answer to 2 decimal places

Solution is to take the natural log of both sides

$$\ln(32^{x-1}) = \ln(28)$$

$$(x-1)\ln 32 = \ln 28 \dots \text{Using } \log(a^b) = b * \log(a)$$

$$x-1 = \ln 28 / \ln 32$$

$$x = \ln 28 / \ln 32 + 1$$

$$x = 1.96$$

Question 16. 2016 P1 Q4 (10 marks):

Given $\log_a 2 = p$ and $\log_a 3 = q$, where $a > 0$, write each of the following in terms of p and q :

(i) $\log_a \frac{8}{3}$

$$p = \log_a 2, \quad q = \log_a 3$$

$$\log_a \frac{8}{3} = \log_a 8 - \log_a 3$$

$$= \log_a (2)^3 - \log_a 3$$

$$= 3 \log_a 2 - \log_a 3$$

$$= 3p - q$$



(ii) $\log_a \frac{9a^2}{16}$.

$$\begin{aligned}\log_a \frac{9a^2}{16} &= \log_a (3a)^2 - \log_a (2)^4 \\ &= 2 \log_a 3 + 2 \log_a a - 4 \log_a 2 \\ &= 2q + 2(1) - 4p \\ &= 2q + 2 - 4p\end{aligned}$$

Question 17. 2014 P1 Q2

Given that $p = \log_c x$, express $\log_c \sqrt{x} + \log_c (cx)$ in terms of p .

We know that

$$\log_c \sqrt{x} = \log_c x^{\frac{1}{2}} = \frac{1}{2} \log_c x = \frac{1}{2}p$$

using the power law for logarithms.

Also,

$$\log_c (cx) = \log_c c + \log_c x = \log_c c + p$$

using the product rule for logarithms.

But $\log_c c = 1$ since $c^1 = c$. Therefore

$$\log_c \sqrt{x} + \log_c (cx) = \frac{1}{2}p + 1 + p = \frac{3p}{2} + 1.$$



Additional Questions

Question 18.

- (i) Let x = Stage number.

There are 4 times as many blue tiles than the Stage number

Blue tiles = $4x$

There are 4 white tiles in every stage. This is a constant and remains 4 no matter what stage number we use.

The total number of green tiles is the square of the stage number.

Number of Green tiles = x^2

The total number of tiles (T) must be the green tiles + blue tiles + white tiles

$$T = x^2 + 4x + 4$$

- (ii) $x^2 + 4x + 4 = 324$

Factorise $(x+2)(x+2) = 324$

$$(x+2)^2 = 324$$

$$x+2 = 18$$

$$x = 16$$

There are x^2 green tiles therefore $16^2 = 256$ green tiles.

- (iii) Mary's kitchen is square. Therefore the length of each side = $\sqrt{6.76} = 2.6\text{m} = 260\text{ cm}$.
Each tile has sides of 20 cm each and $13 \times 20 = 260$. Therefore there are 13 tiles on each side or in each row.
In the first row there are two white tiles and the rest ($13-2=11$) are blue. Therefore this must be stage 11.

$$\text{Green} = x^2 = 121$$

$$\text{Blue} = 4x = 44$$

$$\text{White} = 4$$

Check: The total number of tiles = $121+44+4=169$.

The area of each tile = $0.20 \times 0.20 = 0.04\text{ m}^2$. The total number of tiles needed = $6.76 \div 0.04 = 169$.



Question 19.

1)

200 m Race:

$$y = a(b - x)^c$$
$$y = 4.99087(42.5 - 23.8)^{1.81}$$
$$y = 1000$$

Javelin:

$$y = a(x - b)^c$$
$$y = 15.9803(58.2 - 3.8)^{1.04}$$
$$y = 1020$$

2)

$$y = a(x - b)^c$$
$$1295 = 15.9803(x - 3.8)^{1.04}$$
$$81.0373 = (x - 3.8)^{1.04} = z^{1.04}$$
$$\log z = \frac{\log 81.0373}{1.04}$$
$$z = 68.4343 = (x - 3.8)$$
$$x = 72.2343 = 72.23 \text{ m}$$

3)

$$y = a(b - x)^c$$
$$1087 = 0.11193(254 - 121.84)^c$$
$$\frac{1087}{0.11193} = (132.16)^c$$
$$\log 9711.426 = c \log 132.16$$
$$c = \frac{\log 9711.426}{\log 132.16} = 1.88$$