Future Life Expectancies in Ireland

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ABSTRACT

Mortality forecasts for the Irish population are published following each census by the Central Statistics Office (CSO) as part of their labour force and population projections. The projections rely on identifying and extrapolating past trends in mortality improvements. However, since the calendar year 2011, there has been a significant slow-down in mortality improvements and, in fact, mortality rates observed at ages above 90 years increased in Ireland - a reversal of the long-term trend decline that must cause much unease to public health policy-makers. The recent change in trend poses challenges when forecasting mortality rates. This paper sets out the approach eventually adopted by the CSO in the recent mortality projections, and contrasts it with other extrapolative methods including the increasingly popular stochastic and coherent methods. Comparing the outputs with these models gives a measure of the uncertainty of the future mortality forecasts for Ireland. The mortality projection for Ireland is also compared with the cohort-adjusted approach employed by the Office of National Statistics (UK) for mortality projections for Northern Ireland, Scotland, and England & Wales. We report that there are only minor differences in projected life expectancies, despite the differences in approaches and assumptions used, so we can conclude that the official mortality rates for Ireland (Central Statistics Office, Ireland (CSO, 2018)) and Northern Ireland (Office for National Statistics (ONS, 2017b)) are not inconsistent. Previous CSO mortality projections have been adopted by the actuarial profession in Ireland and others over the last decade for reserving for pension liabilities, for estimating the value of pensions, and to help judge the sustainability of the Social Insurance Fund. This detailed analysis of the CSO's most recent projections, and comparison with other mortality projections for Ireland, will help those considering its adoption for their purposes and gives a measure of the uncertainty surrounding the forecast. We conclude by setting out the implied cohort life expectancy in Ireland, based on the CSO mortality projections, to help individuals' planning for their future lifetime.

Keywords: mortality, life expectancy, Ireland, projecting mortality rates, stochastic mortality models, coherent mortality forecasts, population projection, cohort life expectancy.

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1. Introduction

Shortly following each quinquennial census in Ireland, the Central Statistics Office (CSO) publish population and labour force projections to aid planning of resources for the future needs of the population (e.g. CSO (2018), CSO (2013), CSO (2008)). Projecting the future mortality rates of the population form part of this exercise and, though the ultimate population and labour force forecasts are considerably less sensitive to this assumption than others (such as migration levels and fertility rates), the expert group advising the CSO devote care to this element as, over the last decade, the projections made by the CSO have been widely adopted in applications where future mortality rates are required. So, for instance, professional guidance for actuaries in Ireland when estimating the amount or value of pensions requires allowance to be made for future mortality improvements in line with the CSO rates of mortality improvements (see SAI (2015), SAI (2014), SAI (2008)). Mortality projections have a significant impact on the results in these applications as noted in *The Actuarial Review of the Social Insurance Fund 2015*

> "... mortality improvement rates into the future are projected in line with the CSO Population and Labour Force Projections 2016 - 2046. These population projections allow for a more Irish specific view of the rate of future mortality improvements into the long term – an area of significant judgement – and materially impacting the projections..."
>
> — Department of Employments and Social Protection (2017), p.43

Mortality projections following the 2016 census have recently been published together with a brief outline of the method and parameters adopted (CSO (2018)). Both authors of this paper were members of the expert group advising the CSO, and outline here more fully the factors considered before the basis on mortality projections was eventually adopted. We discuss the key issues as we view them, contrast the official projections with alternative approaches, and provide a measure of the uncertainty in the projections. We conclude by giving estimates of the remaining life expectancy (the 'cohort' life expectancy) of those alive in Ireland today based on how mortality rates are expected to evolve in future years based on the CSO 2018 projections.

The method the CSO apply to projecting mortality rates is unchanged over the last decade, and is described, including a comparison with alternative methods, in Whelan (2008). The projections in 2013, succinctly outlined in CSO (2013), followed the same general methodology but with updated parameters, see Hall (2013a) for a full discussion. The forecasting method used by the CSO is from the popular group of 'targeting methods', where short-term trends in mortality improvements are projected to converge over the following 25 years to the underlying long-term trend of improvement observed in the past. A key issue with the CSO 2018 projections (CSO (2018)) is that short-term trends in population mortality improvements are less clear-cut than previously - it appears that there has been a significant slowing in the rate of improvements since the previous forecasts. However, the pattern of change is very uneven at the older ages in recent years, where, surprisingly, increases in mortality rates were recorded at some ages. Also, the current short-term trends in male and female mortality rates, if used unadjusted in the forecasting methodology, produced forecasts where the gender differential in future life expectancies falls below long-established historic norms. Accordingly, the recent CSO 2018 projections required more judgement in deciding what short-term trend in mortality improvements across the age spectrum and between the genders to input into the forecasting model than the more straightforward data-driven estimates that sufficed in the 2008 and 2013 projections.

The objective of this paper is to set out these and other considerations that helped inform the latest official mortality projections. There are many applications where allowance should be

made for future changes in mortality rates and longevity (e.g. in planning future healthcare needs, in pension planning), some requiring a best estimate approach but others perhaps demanding a more cautious approach (such as establishing the solvency of an annuity or pension provider). So, alongside the CSO 2018 mortality forecasts, we highlight the potential range of future life expectancies using various stochastic models so the probability of life expectancies being above or below a given number can be estimated.

Indeed, the confidence with which life expectancies can be forecast could become a significant policy issue the next time the CSO is due to project the rates in five years' time. The Government commits to an actuarial assessment of life expectancies in 2022, to a study of the ratio between years of life of working and expected years of life in retirement, and "at that point, informed by the review and assessment, a notice period of 13 years will be given in respect of any planned changes to the State pension age before implementation occurs" (Government of Ireland (2018) p.9 and also p.12). We contrast the methods employed and the current range of estimates of projected life expectancies on the island of Ireland made by the Central Statistics Office, by the United Nations new probabilistic model, and by the latest projections from the Office of National Statistics for Northern Ireland. We also survey the demographic and actuarial literature and apply a benchmark stochastic model for forecasting life expectancies and the associated uncertainty to Irish data. We note the extent to which the forecasts changed from the previous time made. Accordingly, we provide three distinct measures of the uncertainty surrounding forecasts of future life expectancies in Ireland: (1) the range of results obtained from different credible modelling approaches applied to Irish data; (2) the confidence bounds to estimates generated by stochastic models applied to Irish and related mortality data; and, (3) the extent to which estimates of future life expectancies in Ireland have changed in recent iterations of the models.

This paper is structured as follows: Section 2 overviews the trends in mortality improvement in Ireland in both the long and short-term, putting them in the context of broader international developments. It highlights a significant slowdown in the rate of improvement since 2011, especially at older ages, so that the previous CSO projections following the 2011 census (CSO (2013)) proved too optimistic in the short-term. Section 3 surveys the wide range of available projection methodologies and subsections consider and critique each main approach in more depth i.e. the CSO approach adopted for the 2018 projections, the ONS approach to forecasting for Northern Ireland adopted in 2017, the Lee-Carter stochastic model applied to Irish data, and the coherent Bayesian stochastic approach applied by the United Nations to Ireland. Section 4 outlines the difference between the period life expectancies forecast by the models and the more relevant cohort life expectancies that estimate the expected remaining lifetime of individuals. Estimates of cohort life expectancies for those living in Ireland are given. The conclusion, in Section 5, summarizes the results and the implications.

2. Historical Trends in Mortality Rates and Life Expectancies in Ireland

2.1. Long-term Trends

A trend of falling mortality rates with the passage of time has been observed in Ireland since the second half of the nineteenth century. The trend declines in mortality rates led to life expectancies at birth increasing by an average 0.26 years for males and 0.30 years for females with the passage of each calendar year over the twentieth century. Mortality improvements over the last century

and longer were not, of course, uniform over either calendar year or year of age. At the start of the last century mortality improvements were more pronounced at the younger ages with little or no improvements discernible at older ages. As the century progressed, improvements were evidenced at all ages and most especially at the older ages in the last few decades (see Whelan (2008) for an overview, Hall (2013b) for an analysis by cause of death and Whelan (2009b), Whelan (2009c) for an analysis of trends at older ages).

Gains in Irish life expectancy came primarily from reductions in infant and child mortality during the first half of the 20th century but gains in the latter half have been due to decline in mortality rates in the final decades of life (most notably from a decline in mortality due to diseases of the circulatory system). This pattern has been called 'the ageing of mortality improvements' and, as Table I illustrates, this pattern, where gains in life expectancy are more pronounced at the older ages, has continued into the early part of the 21st century.

Table I. Gains in life expectancy in Ireland, from birth and age 65 years, by gender, 1926-2015.

	Males			**Females**		
	Gains in Life Expectancy from		Ratio of gains due to	**Gains in Life Expectancy from**		Ratio of gains due to
Period	Birth	Age 65 years	improvements after age 65	Birth	Age 65 years	improvements after age 65
1911-1926	3.8	-0.2	-5.3%	3.8	0.0	0.0%
1926-1936	0.8	-0.3	-37.5%	1.7	-0.3	-17.6%
1936-1946	2.3	-0.5	-21.7%	2.8	0.0	0.0%
1946-1961	7.6	0.6	7.9%	9.5	1.3	13.7%
1961-1971	0.7	-0.2	-28.6%	1.6	0.6	37.5%
1971-1981	1.3	0.2	15.4%	2.1	0.7	33.3%
1981-1991	2.2	0.8	36.4%	2.3	1.4	60.9%
1991-2002	2.8	2.0	71.4%	2.4	1.6	66.7%
2002-2011	3.3	2.3	69.7%	2.5	1.9	76.0%
2011-2015	1.2	0.5	41.7%	0.7	0.3	42.9%

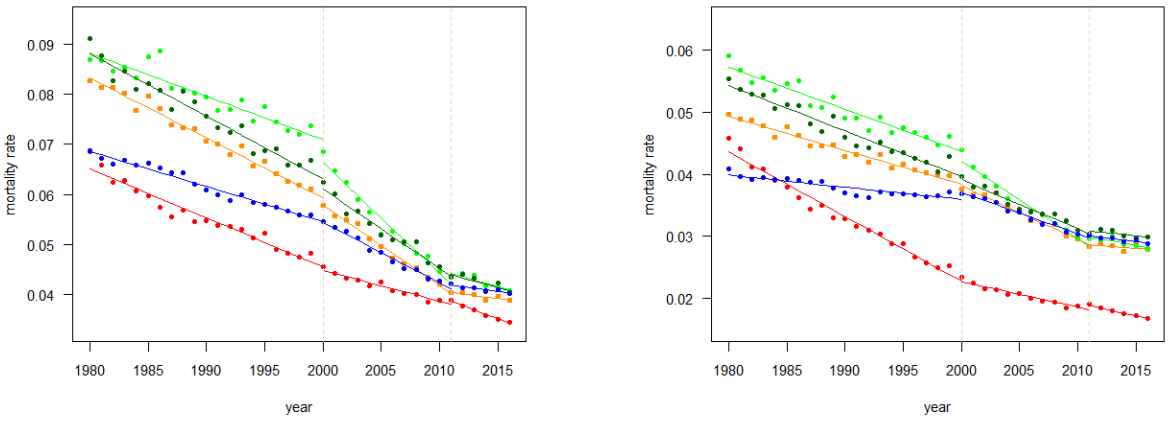
Source: Authors' calculations from figures in Table 3 of CSO (2015).

The broad pattern of mortality improvement over the long term is not unique to Ireland: it is similar in most developed countries. Much of our current understanding of mortality improvements over the twentieth century and, indeed, since early civilisations, is summarised in surveys such as Lancaster's *Expectations of Life: A Study in the Demography, Statistics, and History of World Mortality* (Lancaster (1990)) or Riley's more accessible *Rising life expectancy: a global history* (Riley (2001)). Riley (2001) presents a persuasive case that, in the sweep of human history, mortality reductions can be attributed to six broad (and overlapping) factors: nutrition, wealth and income, behaviour, education, public health, and medicine. The key point is that the mix can be quite different in different countries – especially countries playing catch-up such as many in sub-Saharan Africa - even though the resultant pace of mortality decline has been similar. Recent comparative studies of mortality trends across European countries over the last few decades highlight the increasing homogeneity in mortality improvement patterns leading to a convergence in life expectancies across Western Europe (see, for instance, Avdeev et al. (2011), Meslé (2004), Meslé, Vallin, and Andreyev (2002)). Indeed, Meslé et al. (2002) argue the reason that some, mainly Eastern, European countries do not exhibit such convergence is solely due to behavioural and public health factors, principally due a failure to curb mortality rates from lifestyle diseases. Further studies (such as Klenk, Keil, Jaensch, Christiansen, and Nagel (2016), Leon (2011), Parr, Li, and Tickle (2016), Wilmoth (1998), Wilmoth (2000)) suggest that this observation also holds further afield.

2.2. Short-term Trends

Mortality rates vary significantly over the lifespan, with the mortality rate of a man aged 80 years being about 800-times greater than the mortality rate of a 10 year-old boy. Indeed, according to the latest published Irish life tables (CSO (2015)), current mortality rates imply that there is now a probability of less than 15% of an Irish person dying before their 65^{th} birthday. Accordingly, analysis of trends in mortality rates should concentrate more on trends in mortality rates at older ages, as these are now having a greater impact on future life expectancies.

To enable international comparisons, age-standardised mortality rates are plotted in Figure 1 for ages 65-89 years in Ireland, Northern Ireland, England & Wales, the US, and Japan since 1980. Three different trends are common across all countries: a period of particularly rapid decline in the period 2000-2011, preceded and proceeded by periods of less rapid improvements. Japan is of particular interest as it shows, despite having lower mortality rates over almost the entire period, the trend decline has been at least as steep as the other nations, and steeper since 2011 for both sexes. Life expectancy in Japan is the highest in the world and, with no signs of mortality improvements slowing, humankind is unlikely to be approaching any biological limit to human life as yet (see Oeppen and Vaupel (2002)).



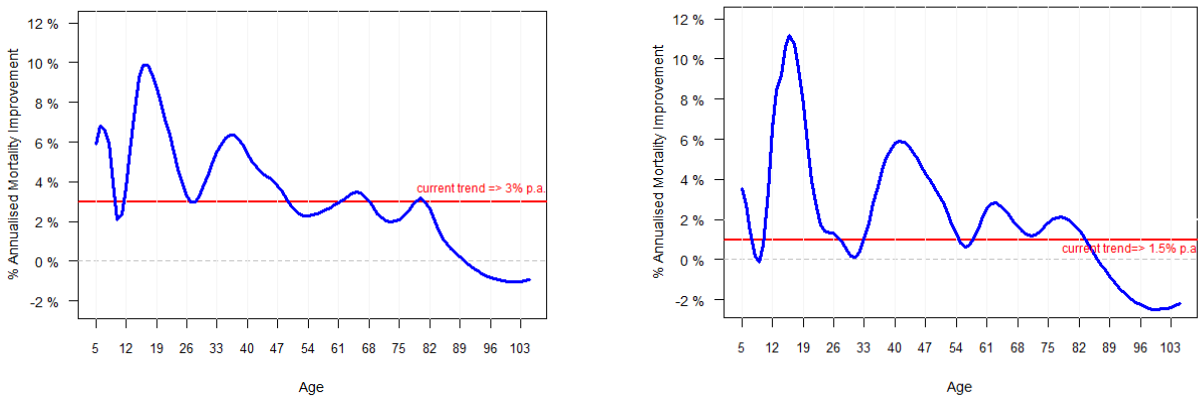
(a) Male **(b)** Female

Figure 1. International age-standardised mortality rates 1980-2016 (ages 65-89 years inclusive) with trend-lines, by gender [—Ireland, —England & Wales, —Northern Ireland, —USA, —Japan] Source: HMD (2018), Pace, Lanzieri, Glickman, and Zupanič (2013), Ahmad et al. (2001).

Figure 1 graphs a selection of a growing body of data that suggests there has been a significant shift in the trend of mortality improvements internationally since about 2011. The change in trend is not entirely accounted for by one-off events causing unusually heavy mortality, such an influenza outbreak or unusual bad weather conditions (see, for example, Adams et al. (2006), Denney, McNown, Rogers, and Doubilet (2013), Ng et al. (2014), Olshansky et al. (2005), Preston, Vierboom, and Stokes (2018), Institute and Faculty of Actuaries (IFoA, 2017)). Analysis of subgroups of populations also report similar findings with for instance, the The Continuous Mortality Investigation of mortality underlying insurance contracts and pension schemes in the UK reporting that average mortality improvements over six years since 2011 have been 0.5% p.a. for males and 0.1% p.a. for females, significantly lower than for any other recent six-year period (C.M.I. (2018)).

The pattern of mortality improvement by age in Ireland over the period 2010 to 2015 is presented

in Figure 2 in greater detail. There is a broad, albeit uneven, pattern of mortality improvements reducing as age increases, with those aged above 90 years (both male and female) recording increasing mortality rates over the period.



(a) Male **(b)** Female

Figure 2. Percentage annual rate of mortality improvement by gender and age, Ireland, 2010-2015
Source: Authors' calculations based on data supplied by the CSO (see CSO (2018) and CSO (2013)).

The recent trend of increasing mortality rates at advanced ages is surprising, as it reverses the trend of slow but constant improvements at these ages over the last half-century (see Whelan (2009b)). There are, of course, issues with estimating mortality rates at these later ages due to age rounding and population mis-estimates (see Whelan (2009a)) but, having experimented with the many ways to overcome these potential problems (e.g., method of near-extinct generations and curve-fitting using the known shape of mortality at these ages), we can report that the adverse pattern remains. This recent trend of mortality rates increasing at older ages must cause unease to public health officials. A more detailed analysis of recent trends at advanced ages in Ireland is given in Naqvi (2019).

Table II, given below, summarises the annual rates of improvement over each quinquennial age group over the last decade, last five years, and last three years ending in 2015. As mentioned earlier, it is more important to estimate improvements in mortality rates at older ages accurately rather than younger ages, as it is at older ages where the vast majority of deaths occur. Accordingly, a better average rate of improvement in mortality to apply is an average weighted by deaths, which is shown in the last row of Table II. The previous mortality projections by the CSO were published in 2013 (CSO (2013)) which projected a continuation of then short-term rate of improvements of 3% per annum for males and 2.5% per annum for females (see Hall (2013a)). Table II shows that, in fact, the weighted rate of improvement since turned out somewhat lower, averaging about 2.6% p.a. for males and 1.6% p.a. for females over the 3 years to 2015 and about 3.0% p.a. for males and 1.5% p.a. for females over the 5 years to 2015.

Table II. Annualised improvement of mortality rates in Ireland over different age groups and periods ending 2015.

	Males			Females		
Age Group	2005-2015	2010-2015	2012-2015	2005-2015	2010-2015	2012-2015
0-4	1.6%	1.7%	-0.3%	3.1%	0.9%	2.0%
5-9	8.0%	6.4%	14.0%	2.4%	2.7%	9.8%
10-14	3.4%	5.2%	6.0%	7.6%	7.1%	13.5%
15-19	6.6%	9.7%	6.2%	8.0%	10.6%	15.9%
20-24	3.7%	6.2%	7.6%	3.6%	2.5%	9.4%
25-29	1.2%	3.0%	1.9%	1.4%	1.1%	2.3%
30-34	2.1%	5.0%	5.7%	0.7%	0.5%	0.6%
35-39	1.8%	6.3%	5.4%	2.2%	4.6%	3.9%
40-44	1.9%	4.7%	5.2%	3.5%	5.8%	6.3%
45-49	2.1%	3.7%	4.8%	3.2%	4.2%	4.5%
50-54	2.1%	2.4%	3.1%	1.9%	2.2%	2.1%
55-59	2.1%	2.5%	2.1%	1.4%	0.7%	-0.1%
60-64	2.7%	3.1%	3.5%	2.4%	2.8%	3.5%
65-69	2.8%	3.2%	2.8%	1.9%	1.9%	1.2%
70-74	2.9%	1.9%	1.8%	1.8%	1.2%	0.3%
75-79	2.9%	2.3%	1.8%	2.4%	2.0%	1.6%
80-84	2.0%	2.1%	1.7%	1.9%	1.2%	1.6%
85-89	0.7%	0.4%	-0.2%	1.2%	-0.3%	0.3%
90-94	0.2%	-0.2%	-0.8%	0.2%	-1.1%	-0.3%
95-99	0.0%	-0.3%	-0.8%	0.2%	-1.1%	-0.5%
100-104	0.0%	-0.2%	-0.5%	0.3%	-0.7%	-0.3%
105-109	-0.1%	-0.1%	-0.2%	0.0%	-0.4%	-0.2%
110+	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Averages:						
10- 89						
Unweighted	2.6%	3.8%	3.7%	2.8%	3.0%	4.2%
10- 89						
Weighted by deaths	2.9%	3.0%	2.6%	2.4%	1.5%	1.6%

Source: Authors' calculations based on data supplied by the CSO (see CSO (2018), CSO (2013), CSO (2008)).

3. Methods to Project Mortality

Projections of mortality rates are typically extrapolative: projections depend on identifying and forecasting trends in mortality rates observed in the past. The evolution of mortality rates over the past in different countries share common features, notably:

1. a near-log-linear decline of mortality rates at any particular age with time, and,
2. the rate of decline of the mortality rate with age diminishes with increasing age.

Extrapolative techniques, generally employed by national statistical agencies and others, find and fit such relationships to past data and project mortality rates assuming the relationships to hold into the future. For a survey of the different approaches to forecasting mortality see, for instance, Booth and Tickle (2008), Pitacco, Denuit, Haberman, and Olivieri (2009), Stoeldraijer, van Duin, van Wissen, and Janssen (2013), Wong-Fupuy and Haberman (2004). Past mortality projections have tended to systematically underestimate mortality trends (Murphy (1995), Oeppen and Vaupel (2002), Keilman (2008), Waldron (2005)) and so understate future life expectancies. This has been

largely due to forecasters predicting a levelling off or slowdown in the rate of mortality improvements while rates of improvement tended, in actuality, to increase.

The extrapolative approach employed by the CSO and other national statistical agencies, though based on relationships found in mortality rates in the past, still requires the input of experts. The forecast mortality rates depend crucially on the time period in the past that is used to determine the short-term rate of improvement input to the model, and a similar dependency exists between the long-term rate input and the long-term period used in the past. So, for instance, if the short-term trend of improvement is estimated for males using the period 2010-2015 then the (weighted) trend would be 3.0%, while if the period used is 2012-2015 then the trend is 2.6% (from Table II earlier). More significantly, if the long-term rate of improvement is estimated over, say the period from 1926 (that is, since Irish Life Table 1) or over the period since 1900 then the former period will give a different (higher) long-term rate of improvement as, in general, mortality improvements have been increasing in the more recent calendar years.

Expert judgement is exercised in the actual rates of improvement decided on, even though it may be later 'objectively' justified by a judicious selection of the periods from which to extrapolate. A second, and related, criticism of extrapolative methods is that expert judgement needs to be exercised also when forecasting mortality rates of subgroups within the same population or for two related populations. For instance, mortality forecasts are done separately for males and females in Ireland and there is an obvious, but not explicitly stated, constraint on how future mortality rates might be allowed diverge between the sexes. In particular, it is difficult to envisage an expert group standing over projections that forecast male mortality rates below female rates, as whatever the observed trends, the resultant relationship between the projected rates for the sexes is inconsistent with gender differentials observed in the past.

Unease with such implicit use of expert judgement in determining acceptable projected mortality rates has led to the development of more explicit, and more data-intensive, extrapolation techniques in the last couple of decades. First, since the seminal work of Lee and Carter (Lee and Carter (1992)), there has been particular interest in building stochastic models of mortality projections that combine future mortality forecasts with probability distributions, so that the probability that rates will be higher or lower than any particular forecast is also part of the output of the model. Second, 'coherent' projection methods have been developed over the past decade that explicitly treat the requirement of limiting the divergence between projected mortality rates of related groups exposed to similar factors influencing mortality by jointly modelling the future mortality of the related groups (Danesi, Haberman, and Millossovich (2015), Li and Lee (2005), Shair, Purcal, and Parr (2017)). Finally, combining both stochastic modelling and coherent projections with a world mortality database, the recent United Nations (UN) forecasts of period life expectancy by country and region use a Bayesian hierarchical model (Raftery, Alkema, and Gerland (2014)), which is one of the more sophisticated and comprehensive implementation of the current art of extrapolative mortality projections. Other projections methodologies, such as the performance-weighted average of many projection models employed recently by Kontis et al. (2017) provide another way to capture the uncertainty about future trends. Reassuringly, the ensemble of 21 projection models for mortality and life expectancy employed in Kontis et al. (2017) produce broadly similar projected life expectancy at birth, country-by-country, to the recent UN forecasts.

In the next several subsections, we outline, discuss, and provide estimates of future life expectancies in Ireland based on several extrapolative techniques, including targeting and stochastic methodologies. In subsection 3.1, we review the CSO approach, used in the previous 2013 projections and the current 2018 projections; we contrast this method and results with those for Northern Ireland published recently by the Office of National Statistics (ONS (2017b)). Then, in subsection

3.2, we describe the Lee-Carter stochastic model, fit it to Irish mortality data, and use it to forecast future life expectancies, together with 95% prediction bounds. Finally, the latest UN projections for life expectancy in Ireland, with their prediction bounds, are also analysed and compared with the CSO projections.

3.1. Targeting Methods to Project Mortality

The 'targeting' method adopted by the CSO since the 2008 projections (CSO (2008)) is a relatively straightforward version of the extrapolative approach: identified short-term trends are forecast over the short-term future and the short-term trend is blended over the future twenty-five years into a long term rate of improvement similar to the rate of improvement observed over the long-term in the past. The Office of National Statistics (ONS (2017b)) forecasts mortality rates separately for Northern Ireland, Scotland, England & Wales in a similar manner, and produces similar forecasts to the CSO, but there are some secondary but important differences (see later). Whelan (2008) considers the CSO approach, the historic patterns in Irish mortality rates, and contrasts it with other popular approaches at that time.

3.1.1. Irish Mortality Projections

Crude Irish mortality rates over the most recent three calendar years are graduated to avoid the adverse effects of random fluctuations, and the resulting graduated rates are taken as the base table for projections (denoted $q_{x,0}$, as the mortality rate at age x in year 0). In the exercise, particular attention is paid to graduating mortality rates at the higher ages, where there are known data issues and where random fluctuations are more material. Graduating at higher ages is done using the Kannisto formula and methods of near-extinct generations (see Whelan (2009b), Whelan (2009c)). The recent CSO 2018 projections were based on the graduated mortality experience over the three calendar years 2014-2016 (so centred on 2015). The data for both deaths and population estimates include all revisions up to January 2018. Recent trends were then studied from analysing the change in mortality rates for each sex at each age over the previous three years, five years, and longer periods.

The method used for projecting mortality rates is to multiply the mortality rate from the base table by a cumulative reduction factor, $CRF(x,t)$, where x denotes age and t denotes the future time in years from the base year, so:

$$q_{x,t} = q_{x,0} \times CRF(x,t) \tag{1}$$

This projection methodology assumes that short-term rates of improvement will converge to a common "target" or long-term rate of improvement at each age and for both genders, by a target year (taken to be the 25^{th} year of projection) and continue to improve at that constant rate thereafter. Accordingly, the cumulative reduction factor is defined recursively as follows:

$$\begin{aligned} CRF(x,1) &= RF(x,1) \\ CRF(x,t) &= CRF(x,t-1) \times RF(x,t) \quad t > 1 \end{aligned} \tag{2}$$

where it is assumed for

CONDITION 1: $x > 100$

$$RF(x,t) = 1 \quad t > 0$$

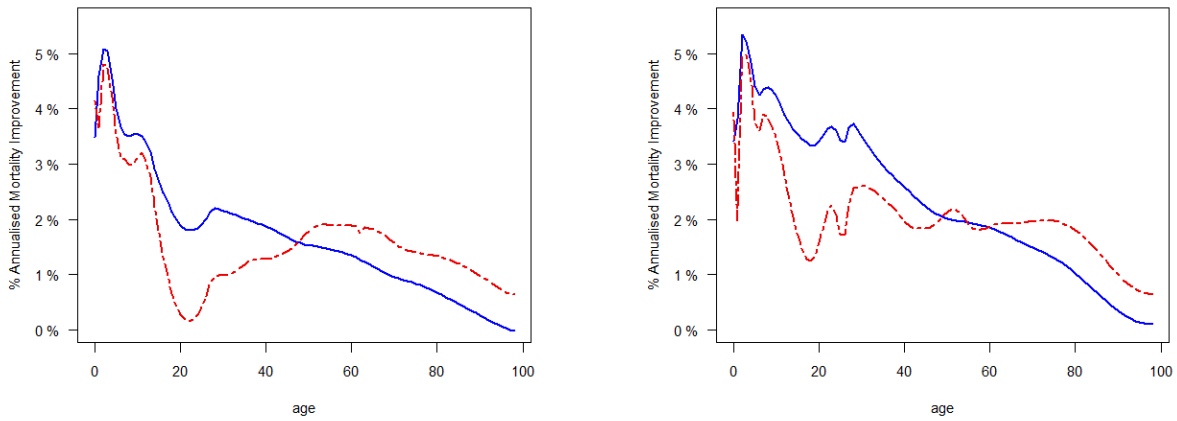
CONDITION 2: $90 < x < 100$.

$$RF(x,t) = RF(90,t) \cdot \frac{100-x}{10} + 1 \cdot \frac{x-90}{10}$$

CONDITION 3: $x \leq 90$.

$$RF(x,t) = \begin{cases} 1 - (\frac{t}{25} \cdot f_{long} + \frac{25-t}{25} \cdot f_{short}) & t < 25 \\ 1 - f_{long} & t \geq 25 \end{cases}$$

For ages between 90 and 100 year, the rate of improvement are derived by linear interpolation between the rates at 90 years and 100 years. The long term rate of improvement, assumed to continue each year from the 25$^{\text{th}}$ projection year, remains unaltered at 1.5% p.a., the same as the two previous projections (CSO (2013), CSO (2008)). This rate is close to the long term rate of both sexes at adult ages over the half century ending 2011 (that is the period before the short-term rate is estimated), as illustrated in Figure 3.



(a) Male **(b)** Female

Figure 3. Annualised fall in Irish mortality per annum, over 50 and 85 years ending 2011, by age [——85 years (1926-2011), - - -50 years (1961-2011)] Source: Authors' calculations based on age-specific mortality rates published by the CSO (see CSO (2012a), CSO (2012b) and CSO (2015)).

The short-term rates of improvements for the previous projections were estimated to be 3.0% p.a. for males and 2.5% p.a. for females based on the average rate of improvement over 4 years to 2010 at each age (see Hall (2013a)). All other parameters were the same as for the current 2018 projections, as summarised in Table III.

Table III. CSO 2013 projection basis.

Base Year: **2010**

Short-term Rates of Improvement

Age	Male	Female
$0-90yrs.$	$3.0\%p.a.$	$2.5\%p.a.$
$91-99yrs.$	estimated by linear interpolation between assumed rate and 0% p.a. at 100 years	estimated by linear interpolation between assumed rate and 0% p.a. at 100 years
$100+yrs$	$0.0\%p.a.$	$0.0\%p.a.$

Long-term Rates of Improvement (from 2036 onwards)

Age	Male	Female
$0-90yrs.$	$1.5\%p.a.$	$1.5\%p.a.$
$91-99yrs.$	estimated by linear interpolation between assumed rate and 0% p.a. at 100 years	estimated by linear interpolation between assumed rate and 0% p.a. at 100 years
$100+yrs$	$0.0\%p.a.$	$0.0\%p.a.$

Source: (CSO (2013)), with further details in Hall (2013a).

The age-specific structure of mortality improvement underwent significant changes by the time of the current 2018 projections, as outlined earlier in Table II. The weighted average rate of improvement over the 5 years to 2015 was 3.0% p.a. for males but only 1.5% p.a. for females. The basis adopted for the CSO 2018 projections is summarised in Table IV.

Table IV. CSO 2018 projection basis.

Base Year: **2015**

Short-term Rates of Improvement

Age	Male	Female
$0-90yrs.$	$2.5\%p.a.$	$2.0\%p.a.$
$91-99yrs.$	estimated by linear interpolation between assumed rate and 0% p.a. at 100 years	estimated by linear interpolation between assumed rate and 0% p.a. at 100 years
$100+yrs$	$0.0\%p.a.$	$0.0\%p.a.$

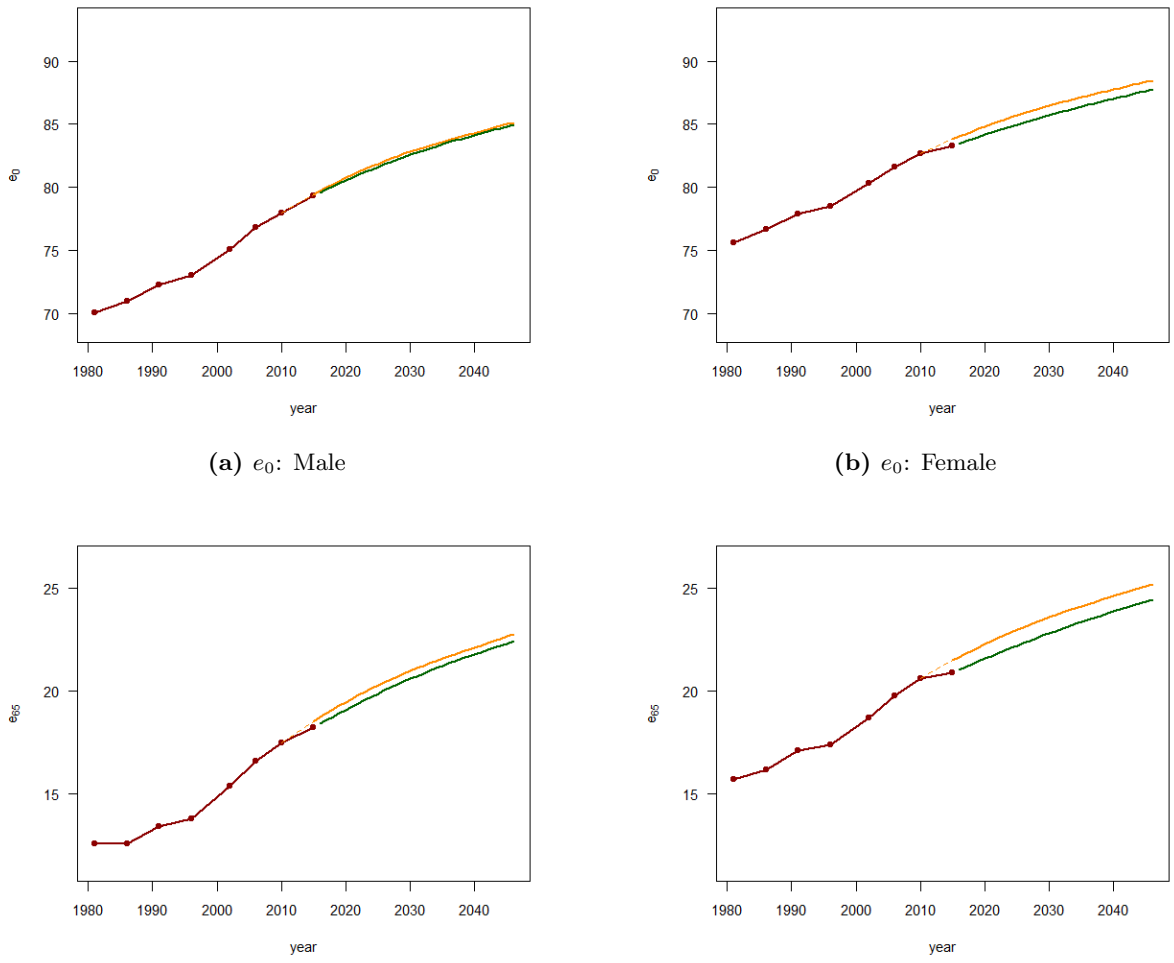
Long-term Rates of Improvement (from 2041 onwards)

Age	Male	Female
$0-90yrs.$	$1.5\%p.a.$	$1.5\%p.a.$
$91-99yrs.$	estimated by linear interpolation between assumed rate and 0% p.a. at 100 years	estimated by linear interpolation between assumed rate and 0% p.a. at 100 years
$100+yrs$	$0.0\%p.a.$	$0.0\%p.a.$

Historically, a difference in life expectancy at birth has favoured females over males by around 2 to 7 years in most countries over most periods (see Kalben (2000)). If we use the weighted average rate of improvement over the 5 years to 2015 of 3.0% p.a. for males and 1.5% p.a. for females then the projected gender differential in life expectancy at birth would breach the lower historical threshold of 2 years from calendar 2036 onwards. It was decided for the 2018 projections to adopt 2.5% per annum as the short-term rate of improvement for males and 2.0% p.a. for females. This

entailed a 0.5% p.a. reduction for both genders from the 2013 projection trend rate. The resultant projection basis ensured that the gender differential in life expectancy at birth is preserved within historic limits (being 2.7 years in the calendar year 2051).

It is of interest to compare projected life expectancies in Ireland under the CSO 2013 and 2018 projection bases, if only to see the impact that changed mortality trends in a five-year period can have on projected life expectancies. In Figure 4, the projected life expectancies from each projection are graphed for future calendar years from birth and at age 65 years, for males and females separately. The impact on observed and projected life expectancies due to the slowdown in mortality improvements over the last few years is obvious, especially so for female life expectancies.



(a) e_0: Male

(b) e_0: Female

(c) e_{65}: Male

(d) e_{65}: Female

Figure 4. CSO projections of period life expectation from birth (e_0) and age 65 years (e_{65}) by projection basis and gender [—Observed, —CSO 2018 projections, —CSO 2013 projections].

The difference in the forecast period life expectancies due to the evolving trends over the five years is summarised below in Table V. Most of the differences, as could be expected, come in estimating life expectancies from age 65 years.

Table V. Projected period life expectancy at birth and at age 65, by gender and CSO projection basis.

	Male Life Expectancy		Female Life Expectancy		Gender difference	
	From Birth	From Age 65 years	From Birth	From Age 65 years	From Birth	From Age 65 years
Projected Values 2030						
CSO 2013 Projections	82.8	21.0	86.5	23.6	3.7	2.6
CSO 2018 Projections	82.6	20.6	85.7	22.8	3.1	2.2
Difference	**-0.2**	**-0.4**	**-0.8**	**-0.8**	**-0.6**	**-0.4**
Projected Values 2045						
CSO 2013 Projections	85.0	22.7	88.3	25.1	3.3	2.4
CSO 2018 Projections	84.8	22.3	87.6	24.4	2.8	2.1
Difference	**-0.2**	**-0.4**	**-0.7**	**-0.7**	**-0.5**	**-0.3**

Finally, we conclude this subsection by noting the sensitivity of projected life expectancies to the parameters in the projection basis used by the CSO. Table VI shows that period life expectancies are more sensitive to the assumptions the longer the forecast period. Perhaps, less obviously, life expectancies at age 65 years are proportionately more sensitive to the projection basis than life expectancies at birth.

Table VI. Sensitivity of life expectancies to key parameters in mortality projection basis.

	Period Life Expectancy In 2030 (years)				Period Life Expectancy In 2045 (years)			
	From Birth	*Change from Central*	From Age 65 years	*Change from Central*	From Birth	*Change from Central*	From Age 65 years	*Change from Central*
Male								
Central Projection Basis	**82.6**		**20.6**		**84.8**		**22.3**	
Initial Decline – Up 1% p.a.	83.5	*0.9*	21.3	*0.7*	85.9	*1.1*	23.2	0.9
Initial Decline – Down 1% p.a.	81.6	*-1.0*	19.9	*-0.7*	83.7	*-1.1*	21.5	-0.8
Long-term Decline – Up 0.5% p.a.	82.8	*0.2*	20.8	*0.2*	85.6	*0.8*	23.0	0.7
Long-term Decline – Down 0.5% p.a.	82.3	*-0.3*	20.4	*-0.2*	84.0	*-0.8*	21.7	-0.6
Female								
Central Projection Basis	**85.7**		**22.8**		**87.6**		**24.4**	
Initial Decline – Up 1% p.a.	86.6	*0.9*	23.5	*0.7*	88.6	*1.0*	25.1	0.7
Initial Decline – Down 1% p.a.	84.8	*-0.9*	22.1	*-0.7*	86.7	*-0.9*	23.6	-0.8
Long-term Decline – Up 0.5% p.a.	85.9	*0.2*	23.0	*0.2*	88.3	*0.7*	24.9	0.5
Long-term Decline – Down 0.5% p.a.	85.5	*-0.2*	22.7	*-0.1*	86.9	*-0.7*	23.8	-0.6

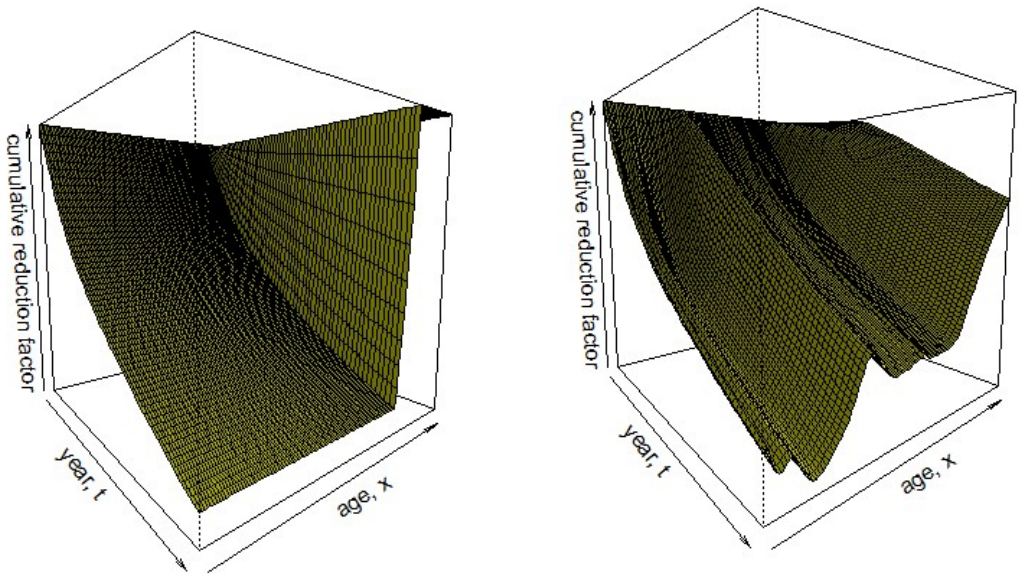
3.1.2. Comparing Irish Mortality Projections with those of Northern Ireland and the UK

The mortality assumptions underlying the most recent populations forecasts in the UK (the 2016-based National Population Projections) are set out in ONS (2017a) and ONS (2017b). Similar, to the approach by the CSO, the ONS use a targeting approach, blending current short-term rates of improvement by age and gender to long-term uniform rates over the next 25 years. Projections are done overall for the UK and by each constituent nation (Northern Ireland, Scotland, England, and Wales), with the parameters for current trends used for Scotland being different to the other nations, reflecting its different pattern of mortality improvements over the period 1961-2015.

The key assumptions in the mortality projections for Northern Ireland and the UK overall can be summarised as:

- Long-term rate of improvement after 25 years: 1.2% p.a., for those aged under 92. For those aged between 92 and 110 the rate declines from 1.2% p.a. to 0.1% p.a. and remains at 0.1% p.a. for those aged over 110 years.
- Currently observed short-term rates of improvement, separately estimated by age and sex, were used for the first year of projection and were assumed to converge to the long-term rates over a 25 year period. Current rates of improvement were all positive and higher for males across most ages (and all ages over 50 years). Convergence from current rates of improvement to the long-term rates are assumed at the same pace for males and females, and for those born between 1940 and 1960 the convergence is by cohort.

So the reduction in mortality assumed under the two approaches are different, and perhaps the rates used are best compared in graphical (Figure 5) and tabular form (Table VII), as given below.



(a) CSO method - Ireland **(b)** ONS method - UK (excl. Scotland)

Figure 5. Profile of cumulative reduction factor, $CRF(x,t)$, against age x and future year t according to method.

The more significant differences in the projection methodologies employed is that Irish mortality rates at higher ages are projected to fall more rapidly that the ONS projections for the UK excluding Scotland. From 25 years onwards the Irish mortality rates up to age 90 are projected to fall by 1.5% p.a. while in the UK the corresponding assumed rate 1.2% p.a.. Another difference in the forecasting approaches is that the UK projections allow for a cohort effect. Indeed, forecasting mortality rates by cohort has been a feature of official projections in the UK since a pattern of improvement by birth year was observed during an exploratory analysis of past trends in 1995 (Office of Population Censuses & Surveys (1995)). In particular, a so-called "golden cohort" was identified as those born between calendar years 1923 and 1938, that had higher rates of improvement than previous and subsequent generations.

Table VII. Assumed percentage reduction in mortality rates rates by selected ages and calender periods.

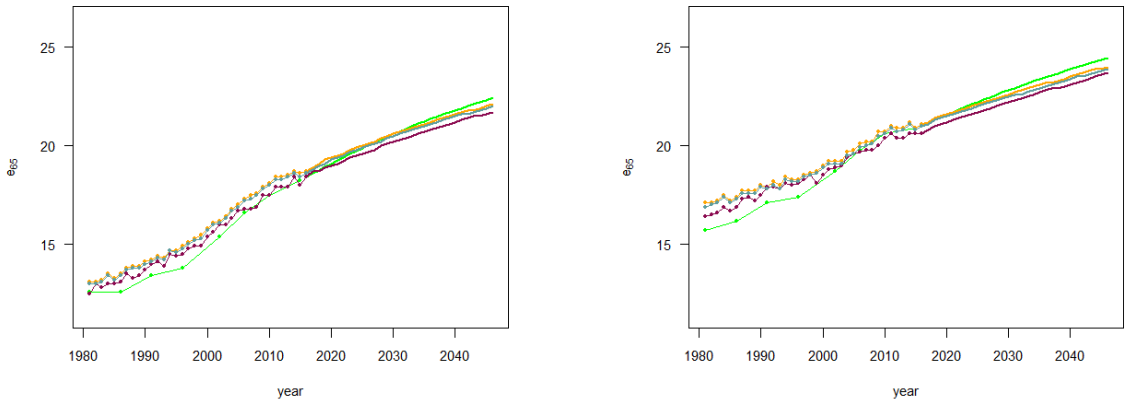
	2015-2016	2016-2017	2030-2031	2030-2031	Cumulative 2015-2040 (25 year)	Cumulative 2016-2041 (25 year)
Age last birthday	Ireland[a]	UK (excl. Scotland)[b]	Ireland[a]	UK (excl. Scotland)[b]	Ireland[a]	UK(excl. Scotland)[b]
Male						
0	2.5%	2.5%	1.9%	1.7%	39.4%	36.4%
5	2.5%	3.6%	1.9%	2.0%	39.4%	43.8%
10	2.5%	3.2%	1.9%	1.9%	39.4%	41.7%
30	2.5%	3.0%	1.9%	1.8%	39.4%	40.3%
50	2.5%	2.0%	1.9%	1.5%	39.4%	32.3%
60	2.5%	1.8%	1.9%	1.5%	39.4%	32.3%
70	2.5%	2.3%	1.9%	1.5%	39.4%	30.4%
80	2.5%	2.0%	1.9%	1.4%	39.4%	31.9%
90	2.5%	0.9%	1.9%	1.4%	39.4%	29.2%
Female						
0	2.0%	2.6%	1.7%	1.7%	35.5%	37.2%
5	2.0%	3.2%	1.7%	1.9%	35.5%	41.7%
10	2.0%	2.8%	1.7%	1.8%	35.5%	38.7%
30	2.0%	1.3%	1.7%	1.2%	35.5%	27.0%
50	2.0%	1.9%	1.7%	1.5%	35.5%	32.1%
60	2.0%	1.5%	1.7%	1.3%	35.5%	29.2%
70	2.0%	1.8%	1.7%	1.3%	35.5%	28.2%
80	2.0%	1.8%	1.7%	1.3%	35.5%	29.4%
90	2.0%	0.4%	1.7%	1.3%	35.5%	26.2%

[a] Base year 2015; [b] Base year 2016.

There ensued a debate in the actuarial literature as to whether forecasting is better done incorporating year of birth alongside age and calendar year, with arguments in favour of using such cohort projections outlined in Richards (2008), Richards et al. (2007), Willets (2004), Willets et al. (2004). However, the pattern was less convincing in Irish data (see Whelan (2008)). Whelan (2009a) argued that the pattern in the UK could well be attributed to data-mining, as the hypothesis of a cohort effect was prompted by the data, which was then used to verify the hypothesis and, as such, could be an unreliable pattern to project. Evidence was provided that even the Great Famine in Ireland did not appear to have produced a discernible cohort pattern in mortality in the generations born before, during, or after it. Recent mortality data in the UK has shown that the "golden cohort" no longer appear to experience significantly higher rates of improvement than other generations so mortality is no longer projected by cohort for this group (ONS (2017a)). However, UK forecasters still project by cohort for those born between the calendar years 1940 and 1960.

Despite the differences in short-term and long term trends assumed, and the method used to converge the rates over the next 25 years, the recent mortality projections for Ireland, Northern Ireland, and the rest of the UK are surprisingly close as illustrated in Figure 6. The different models for Ireland and Northern Ireland forecast life expectancy at age 65 to be within one year of one another out to 2050 (that is an initial difference of 0.2 years and 0.3 years for males and females respectively in 2015, is projected to rise to 0.4 years for males and 0.6 years for females in 2030, and further increase to 0.7 years for males and 0.8 years for females in 2045).

Another element that the experts advising on the UK projections and those advising on the projections for Ireland did not agree on was the long-term rate of improvement in mortality – that is the rate of improvement after 25 years in the future, where 1.2% p.a. was used in the UK central assumption and 1.5% p.a. in the Irish assumptions.



(a) Male **(b)** Female

Figure 6. Period life expectation at age 65 years (e_{65}) by gender and country [—Ireland, —England & Wales, —Northern Ireland, —United Kingdom].

This is a key parameter in forecasting (see Table VI earlier). The differences are due to analysing different periods in the past and using different weights to average the observed rates of improvement. ONS (2017a) states that the age-standardised rates of improvement from 1961 to 2014 (a period of 53 years) was 1.6% p.a. for males and 1.3% p.a. for females; but was around 1.4% p.a. for both sexes over the last three-quarters of a century and was about 1.2% p.a. for both sexes over the 20$^{\text{th}}$ century in the UK. Whelan (2008) looks at the patterns for Ireland since 1926 (Irish Life Table 1) and shows how it varies by age and, similar to the UK over the same period, suggests 1.5% p.a. as reasonable for all ages up to age 90 years.

It is enlightening to see experts in other countries having similar issues with agreeing a long term rate for mortality improvements. For over two decades now, there has been a heated debate between the Office of the Chief Actuary in the United States, who periodically investigates the financial soundness of the US social security system, and an advisory panel of experts as to what is a reasonable assumption on the long term rate of mortality improvements (as like Ireland and UK, US projections use a single long term rate to which all age-and-sex specific rates are assumed to converge 25 years in the future). Future mortality, especially at older ages, is a key driver of the cost of maintaining the US social security (that is, the Old-Age and Survivors Insurance Trust Fund and the Disability Insurance Trust Fund) and this assumption has is one of the most debated, as the most recent report states

> "...No other assumption has been the subject of a more persistent and unresolved disagreement between the Trustees and successive Technical Panels than that of the assumed ultimate rate of improvement in mortality rates..."
>
> —Technical Panel on Assumptions and Methods (2015)

The Technical Panel argue that long-term mortality improvements should be 1% p.a. (the 2011 Technical Panel suggested 1.25% p.a.) while the Office of the Chief Actuary assumes 0.71% p.a.. The gap between the two has been narrowing over the last two decades as the Office of the Chief Actuary has increased its estimate.

3.2. Stochastic Methods to Project Mortality

3.2.1. Lee-Carter Model

Lee and Carter (1992) is a seminal paper in stochastic mortality forecasting, where point projections of mortality rates are accompanied by prediction intervals that give a measure of their reliability based on the underlying probability model. The relative simplicity of the model, coupled with early success, has ensured that even now, a quarter of a century later, the Lee-Carter model or one its subsequent adaptations remains a benchmark against which other stochastic models are compared (Booth and Tickle (2008), Macdonald, Richards, and Currie (2018), Stoeldraijer et al. (2013)). In the original model, the central mortality rates for age x at time t (denoted $m_x(t)$) are assumed to have the following structure

$$\ln m_x(t) = \alpha_x + \beta_x \cdot \kappa_t + \epsilon_{x,t} \tag{3}$$

where the α_x, β_x are age-specific parameters, κ_t describes the trend in the mortality rate over time (the so-called mortality index), and $\epsilon_{x,t}$ are independent, identically distributed normal random variables with zero mean, and the constraints to ensure a unique solution generally being

$$\sum_t \kappa_t = 0 \quad \sum_x \beta_x = 1$$

Mortality projection under the Lee-Carter method requires only the extrapolation of the mortality index, κ_t, since α_x, β_x are estimated from past data and held constant for the duration of the projection. The β_x measures the sensitivity at each age to changes in the overall mortality index. So, for projection purposes, this can be seen as a single parameter model based on κ_t, an underlying constant exponential rate of decline which is modified at each age by the β_x coefficient. A point to be borne in mind when interpreting the forecast rates and their uncertainty, is that the estimated β_x at high ages is low as, in the past, higher ages have experienced relatively lower mortality improvements. The uncertainty in future mortality rates in the model is proportional to β_x, which can lead to uncertainty being very low for high ages.

Lee and Carter (1992) report that the mortality index κ_t is approximately linear for the United States over the period 1900-1987, and several sub-periods studied and, excluding the flu epidemic of 1918, the variance of κ_t also appears constant. The stability of κ_t over long periods in the past gave them confidence to base predicted future mortality rates on their model. The evolution of κ_t over the future was modelled as a random walk with constant drift and variance (fitted to past values), and extrapolated. Their model predicted period life expectancy of a person born in 2065 in the US would be about 10 years higher at 86 years, with a 95% prediction band of (80.45 years, 90 years) at a time when the US Government Actuary was predicting just 80.45 years.

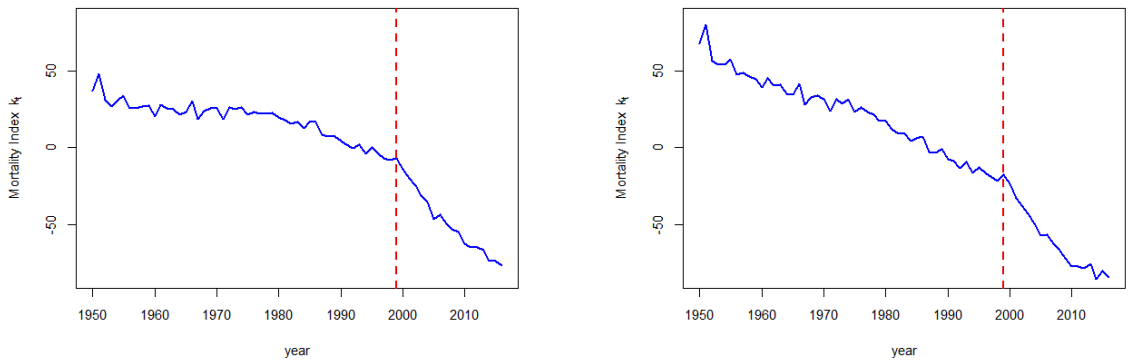
The Lee-Carter model essentially just relies on a near-log-linear decline of mortality rate at any particular age with time and, as such a pattern is evident in most countries, other demographers applied the model to other countries (Tuljapurkar, Li, and Boe (2000)). So, the Lee-Carter model became widely used in forecasting mortality rates and their associated uncertainty. In fact, the Lee-Carter model can be seen as the stochastic version of the method used by the CSO in mortality projections prior to its adoption of the current method (see Whelan (2008)). There have been developments of the original Lee-Carter model. Booth, Hyndman, Tickle, and De Jong (2006) compare the performance of four extensions to the original model, using data from 1986, and report no significant differences in forecast accuracy for life expectancy, but some are more accurate in estimating mortality rates. More recent extensions (such as Cairns et al. (2009), Renshaw and

Haberman (2006)) introduce additional terms to deal with the so-called cohort effect postulated to exist in the UK and elsewhere (see earlier).

One key issue when applying the Lee-Carter model, or one of its more recent extensions, to forecasting is the stability or otherwise of the observed trend of κ_t over past periods. Recent empirical studies report that the mortality index estimated depends to high degree on the past period studied and, in many countries over the last half-century, there is evidence of structural breaks in the historic κ_t series. Fitting the Lee-Carter model and testing for structural changes in estimated mortality indices in the period 1950-2006 for 18 developed countries, Coelho and Nunes (2011) detected the presence of significant structural change in the mortality development of males, coincident with an accentuated decline in the overall rate of mortality for almost every country, including Ireland (where a break was identified in calendar year 1999). Similar evidence supporting structural change in female mortality development has been reported for only for a few countries, but those countries include Ireland (with a break also identified in calendar year 1999). It should be noted that Coelho and Nunes (2011) considered only the possibility of a single structural break during the period of the data. O'Hare (2012) studies extensions to the Lee-Carter model, including extensions to deal with the postulated cohort effect, and also reports structural breaks in the mortality index in several countries over the period 1950-2000.

3.2.2. Lee-Carter Model applied to Irish Data

The empirical findings, as noted above, caution on the use of the Lee-Carter model, and its more recent variants, to forecast mortality rates in Ireland, as the forecast rates will depend on the past period modelled. We fit the Lee-Carter model to male and female mortality rates over the period 1950-2016 and plot the estimated κ_t in Figure 7 below.



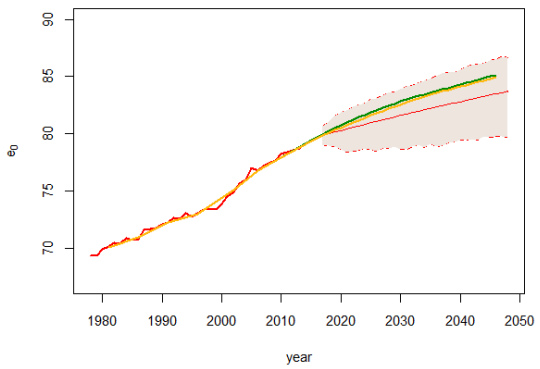
(a) Male **(b)** Female

Figure 7. Mortality Index, κ_t, for Ireland 1950-2016 from fitting the Lee-Carter model by gender [—mortality index, - - -structural break].

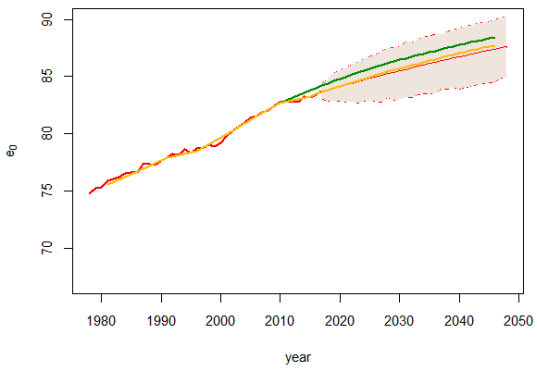
Consider Figure 7 and the fitted κ_t for males (the same comments hold for females). We see a change of slope over the period, with the slope of κ_t over the period 1950-1999 being considerably lower that the slope of κ_t from 1999 to 2016. Using the data on κ_t since 1999 to estimate the drift and variance of the random walk for future projections, we estimate a much faster fall in mortality over future time than using the data 1950-1999 or since 1950. Indeed, this result is typical for most

developed countries as mortality improvements have tended to accelerate in recent decades (see, for instance, Coelho and Nunes (2011)). The conclusion is that the rate of change of mortality projected in the future using the Lee-Carter model depends on the past period selected. Indeed, some researchers (such as Booth, Maindonald, and Smith (2002), Denuit and Goderniaux (2005)) suggest selecting a best 'fitting' period to ensure linearity of the trend component and extrapolating from that. Nonetheless, it is of interest to compare forecasts made by the CSO based on a targeting approach, to those made under the Lee-Carter approach and its associated prediction intervals.

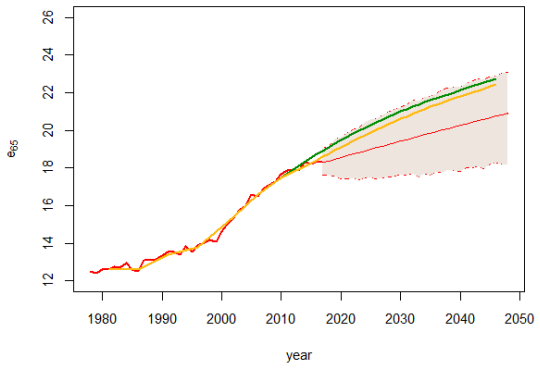
Figure 8 graphs the projections of life expectancy at birth and at age 65 by each future calendar year generated by the unmodified Lee-Carter forecast model when fit to Irish mortality rates over the period 1980 to 2016, together with their 95% prediction interval.



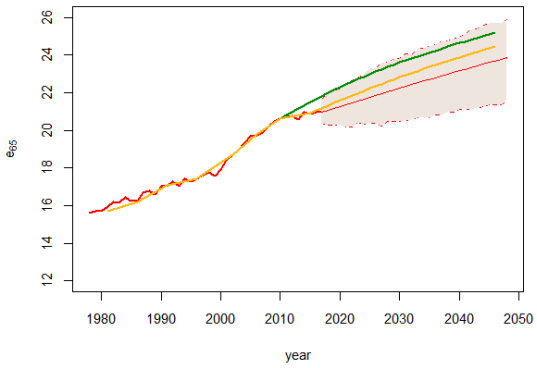
(a) e_0: Male



(b) e_0: Female



(c) e_{65}: Male



(d) e_{65}: Female

Figure 8. Lee-Carter projections, with 95% prediction interval, versus CSO projections of period life expectation from birth (e_0) and age 65 years (e_{65}) by gender [—Lee-Carter (unmodified) projections, —CSO 2018 projections, —CSO 2013 projections].

In Figure 8, the corresponding life expectancies forecast by the CSO in the 2013 and 2018 projections are also shown. The CSO 2013 projections can be interpreted as projections allowing for accentuated mortality decline from 2000, while the 2018 projections can be interpreted as

projections incorporating a further trend change, i.e. incorporating the recent attenuation in rates of mortality improvement. In both cases, median life expectancy projections produced by the CSO targeting-based approach result in higher life expectancy outcomes relative to the (anticipated underestimated) outcomes of the unmodified Lee-Carter forecast model, with the discrepancy being more pronounced for life expectancy at age 65 for males. The Lee-Carter model also forecasts an unchanging gender differential in life expectancy at birth, and a slightly increasing gender differential in life expectancy at age 65, contrary to recent trends of a reduction of the gender differential.

3.2.3. Coherent Forecasting

One issue with models, stochastic or otherwise, that treat populations separately is that forecasts of mortality for either sub-groups within the population or of other related populations can produce inconsistencies in the long-term (Hyndman, Booth, and Yasmeen (2013)). Coherent methods seek to overcome this issue so that projections for related populations maintain historic relationships, e.g. differences in mortality by gender within a single population can be expected to persist within observed limits in the future and projections for similar countries should not differ radically. Full joint modelling has been considered in the Li-Lee method (Li and Lee (2005)), an adaptation of the Lee-Carter method. This method limits the divergence of projections calculated for separate groups by using two components: a factor common to the entire population and another factor specific to each sub-population. The Li-Lee method is based on the following extension to Lee-Carter model

$$\ln m_x(t,i) = \alpha_{x,i} + \beta_{x,i} \cdot \kappa_{t,i} + B_x \cdot K_t + \epsilon_{x,t,i} \tag{4}$$

where the change in mortality over time described by new term $B_x \cdot K_t$ is the "common" factor for each sub-population. The term $\beta_{x,i} \cdot \kappa_{t,i}$ denotes the specific factor of i^{th} sub-population which allows for differences in the rate of change in sub-population i's death rates and the rate of change implied by the common factor. Alternatively, Jarner and Kryger (2011) considers joint modelling of a population's mortality with a larger reference population. Other approaches can also be found - see Shair et al. (2017) for an evaluation of two more recent coherent models.

Apart from being studied in academic literature, the coherent multi-population approach has recently found its way to official population projections in the Netherlands and Canada. Moreover, recent work has sought to constitute coherent forecasting within a Bayesian paradigm. That is, to say for an unknown quantity θ and sample information x, the likelihood function $L(x|\theta)$ provides empirical information on θ (being the probability of observing the sample given θ). The prior distribution $\pi(\theta)$ represents the initial uncertainty on θ. Bayesian inference on θ is made in terms of the posterior distribution $\pi(\theta|x)$, where

$$\pi(\theta|x) \propto \pi(\theta) \cdot L(x|\theta) \tag{5}$$

Essentially, a Bayesian framework allows knowledge and opinions to be expressed in terms of a prior distribution, which may be transformed to the posterior distribution, $\pi(\theta|x)$, by incorporating empirical evidence, $L(x|\theta)$.

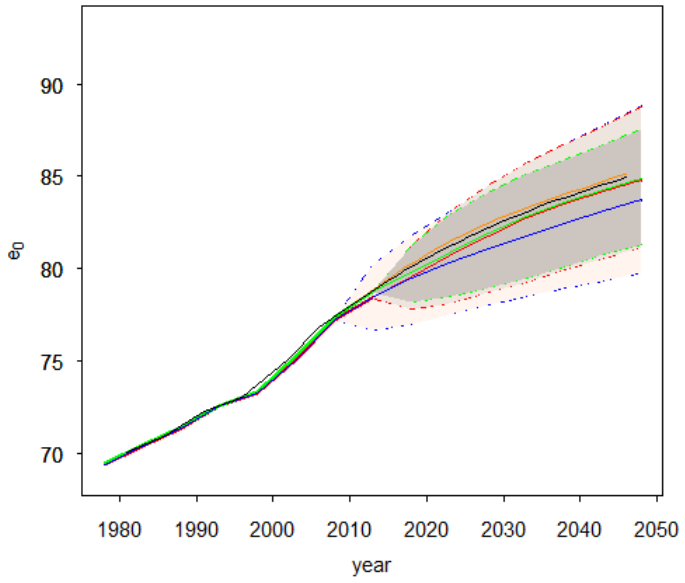
Several Bayesian treatments of mortality projections have been proposed by many authors (Czado, Delwarde, and Denuit (2005), Girosi and King (2008), Kogure, Kitsukawa, and Kurachi (2009), Raftery, Li, Sevcikova, Gerland, and Heilig (2012), Raftery, Chunn, Gerland, and Sevcikova (2013)). Girosi and King (2008) developed a Bayesian framework that incorporate covariates to improve mortality projections, by pooling information from similar cross-sections, e.g. age-groups,

countries. More recently, a sophisticated Bayesian model has been used by the United Nations to predict the future paths of male and female period life expectancy for each country in a coherent manner (Raftery et al. (2014)). The Bayesian framework allows the experience of another population – or, indeed, all other populations - to be readily incorporated into the modelling process by adjusting the parameters of the prior distributions.

3.2.4. Coherent (Bayesian) Forecasting – the recent UN model for Ireland

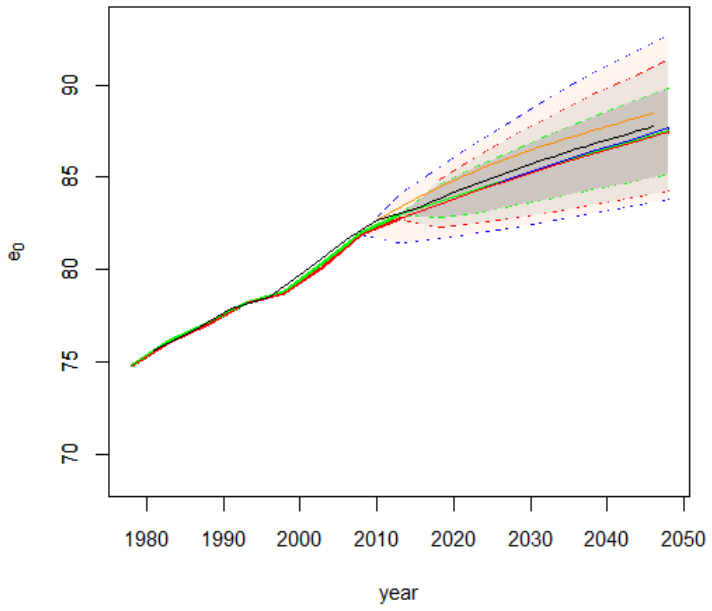
The UN Population Division issued stochastic population projections for the first time for all countries in the world in 2014 (Bijak et al. (2015)). Mortality forecasts underlying these projections were accomplished using a stochastic Bayesian hierarchical model with gains in life expectancy at birth forecast using a deterministic double logistic function with parameters drawn from a common world population (Raftery et al. (2014)) and then male life expectancies were derived from female life expectancies by projecting the gap between the sexes. The UN forecasts in a stochastic and coherent manner the life expectancies for 159 countries, comprising about 90% of the world's population (so excluding some 38 countries with AIDS epidemics because of their very different mortality patterns and 30 countries with populations under 100,000).

It is of interest to contrast the CSO mortality projections for Ireland with the latest UN forecasts. In Figure 9, we graph the life expectancy at birth under both projection approaches, including the 95% prediction intervals of the UN approach. It should be noted that the UN adjusted their standard model for Ireland as it found that the rate of mortality improvement since 1950 was out of line with similar countries and so adjustments were made to the default projection trajectory (U.N.(DESA) (2015) pp. 26-27).



(a) e_0: Male

Figure 9. UN projections versus CSO projections of period life expectation at birth (e_0) by gender and projection year [—2017 UN projections, —2015 UN projections, —2012 UN projections, —CSO 2018 projections, —CSO 2013 projections]. *contd. overleaf*



(b) e_0: Female

Figure 9. UN projections versus CSO projections of period life expectation at birth (e_0) by gender and projection year [—2017 UN projections, —2015 UN projections, —2012 UN projections, —CSO 2018 projections, —CSO 2013 projections].

The CSO predict a higher life expectancy at birth in 2030 at 0.3 years higher for males and 0.5 years higher for females, increasing to 0.6 years for females and remaining unchanged for males in 2045 (see Table XII later). It is notable that while UN median projections of life expectancy for females have remained stable over the projections years, greater variability is evident in case of males. UN median projections of life expectancy at birth for males have come to be more aligned, since the 2012 iteration, with those produced by the targeting approach - this may be due to greater coherence between genders being imposed within the UN model in later iterations.

Importantly, the gender differential in life expectancy at birth is projected to decrease by both models, and is closely matched in 2030 and 2045. The difference between the models in life expectancy at at age 65 in 2030 is 0.7 years for both males and females, increasing to 0.8 and 0.9 years for males and females respectively in 2045. For females, UN median estimates of life expectancy at birth and age 65 present some challenges - the estimates generated are lower than the estimates produced by the unmodified Lee-Carter model (see earlier).

4. Cohort Life Expectancy in Ireland

The latest published population life table for Ireland, Irish Life Table 16 (CSO (2015)), provides estimates of the 'period' life expectancy at different ages, for both males and females, which serves as a useful tool to make comparisons of trends over time, and between geographical areas. However, the period life expectancy does not estimate reliably how much longer an individual might survive on average, as the Background Notes to Irish Life Tables No 16 make clear

"...Period expectation of life at a given age for 2010-12 is the average number of years

a person would live if he or she experienced age-specific mortality rates for that time period throughout his or her life. It is therefore not the number of years someone of that age could actually expect to live because death rates are likely to change in the future. . . "

— CSO (2015), first paragraph of Background Notes

The cohort approach to life expectancy directly addresses the problem of how long an individual, at a particular age, can be expected to live (on average) in the future. The cohort life expectancy is estimated by adjusting recently experienced mortality rates at each age by projecting future changes to these mortality rates as the individual ages. So, for example, a girl aged 5 years now will be aged 55 years in five decades' time so, in estimating the cohort life expectancy, the current mortality rate of a 55-year-old woman is adjusted to reflect how that mortality rate is expected to change over the next half-century. Projected mortality rates are estimated for each future age at each future period and these projected mortality rates are then used in the calculation of the cohort life expectancy (rather than the historic mortality rates as used to calculate the period life expectancy).

The mortality projection method used by the CSO in population and labour force projections can be applied to estimate the remaining cohort life expectancy for a person alive in Ireland at the current time. We have estimated the period and cohort life expectancies in Ireland in the calendar year 2020. Such cohort life expectancies have not been published before, despite their importance to an individual planning for the future, such as helping to estimate how much to save for retirement. Irish period and cohort life expectancies on the CSO mortality projection basis used in CSO (2018) are shown at birth and each decennial age in Table VIII, and are set out in full in Appendix A. It can be seen that there are substantial differences between cohort and period life expectancy due to expected improvements in mortality over future time periods.

Table VIII. Projected period and cohort life expectancies in 2020 in Ireland from CSO 2018 projection basis, by gender and at selected ages.

	Males			**Females**		
Age in 2020	Period LE in 2020	Cohort LE in 2020	Gap	Period LE in 2020	Cohort LE in 2020	Gap
0	80.5	90.4	9.9	84.2	92.7	8.5
10	70.9	79.6	8.7	74.4	82.1	7.7
20	61.0	68.6	7.6	64.5	71.2	6.7
30	51.3	57.7	6.4	54.6	60.3	5.7
40	41.7	46.8	5.1	44.8	49.4	4.6
50	32.2	36.1	3.9	35.2	38.6	3.4
60	23.3	25.9	2.6	26.0	28.4	2.4
70	15.2	16.6	1.4	17.4	18.8	1.4
80	8.4	9.1	0.7	9.9	10.5	0.6
90	4.0	4.1	0.1	4.7	4.8	0.1
100	1.8	1.8	0.0	2.1	2.1	0.0

Source: Authors' calculations.

Table IX shows how estimates of period and cohort life expectancies in the calendar year 2020 have changed from the previous estimates five years ago to the current CSO estimates.

Table IX. Selected CSO projected period and cohort life expectancies in 2020, by gender and projection basis.

	Males			Females		
Projection Basis	Period LE in 2020	Cohort LE in 2020	Gap	Period LE in 2020	Cohort LE in 2020	Gap
From Birth						
2013 Projections	80.5	90.5	10.0	84.8	93.2	8.4
2018 Projections	80.5	90.4	9.9	84.2	92.7	8.5
From age 65 years						
2013 Projections	19.5	21.5	2.0	22.3	24.2	1.9
2018 Projections	19.1	21.1	2.0	21.6	23.4	1.8

Source: Authors' calculations.

Finally, it is of interest to compare estimates of cohort life expectancies by the CSO method, with those of the UN for Ireland and those of the ONS for Northern Ireland.

The UN cohort life expectancy at age 65 estimates for both males and females have been calculated from latest available UN life table data. UN life table data[1] is presented in an abridged form in roughly 5 year age groups (up to end age interval 85+ years), each by quinquennial period from 1950 to 2100; the survivor function, l_x, is also available separately in similar form but with end age interval 100+ years. Several methods exist to extricate cohort life expectancies from such available abridged life table data, including polynomial interpolation, osculatory interpolation, cubic spline interpolation. By using osculatory interpolation, namely Karup-King's third difference method (King (1914), Siegel and Swanson (2004)) and applying further cohort-wise interpolation by ordinary least squares method with yearly steps, we constructed cohort life tables from the available data published by the UN. In Table X we set out the estimated cohort life expectancy at age 65 for males and females and compare the extent of differences between the projection methods.

Table X. Projected period and cohort life expectancies at age 65 in 2020, by gender and projection method.

	Males			Females		
Method	Period LE in 2020	Cohort LE in 2020	Gap	Period LE in 2020	Cohort LE in 2020	Gap
From age 65 years						
Target Method - CSO	19.1	21.1	2.0	21.6	23.4	1.8
Target Method – ONS for Northern Ireland	19.0	20.7	1.7	21.2	22.8	1.6
Coherent Method (Bayesian) - UN	18.5	20.1	1.6	21.1	22.4	1.3

Finally, we conclude by indicating, in Table XI, the sensitivity of the cohort life expectancies estimated using the CSO approach to changes in the parameters for short-term and long term rates of mortality decline.

[1] Available UN life table functions are $m_{x,n}, q_{x,n}, p_{x,n}, l_x, d_{x,n}, L_{x,n}, S_{x,n}, T_x, e_x, a_{x,n}$

Table XI. Sensitivity of estimates of cohort life expectancies estimated under the CSO approach to changes in the parameters for short-term and long term rates of mortality decline, by gender.

	Male Cohort Life Expectancy In 2020 (years)				**Female Cohort Life Expectancy In 2020 (years)**			
	From Birth	*Change from Central*	From Age 65 years	*Change from Central*	From Birth	*Change from Central*	From Age 65 years	*Change from Central*
Central Projection Basis	**90.4**		**21.1**		**92.7**		**23.4**	
Initial Decline – Up 1% p.a.	91.2	*0.8*	21.9	*0.8*	93.4	*0.7*	24.2	*0.8*
Initial Decline – Down 1% p.a.	89.5	*-0.9*	20.3	*-0.8*	92.0	*-0.7*	22.7	*-0.7*
Long-term Decline – Up 0.5% p.a.	92.5	*2.1*	21.5	*0.4*	94.5	*1.8*	23.8	*0.4*
Long-term Decline – Down 0.5% p.a.	87.9	*-2.5*	20.8	*-0.3*	90.5	*-2.2*	23.1	*-0.3*

5. Conclusion

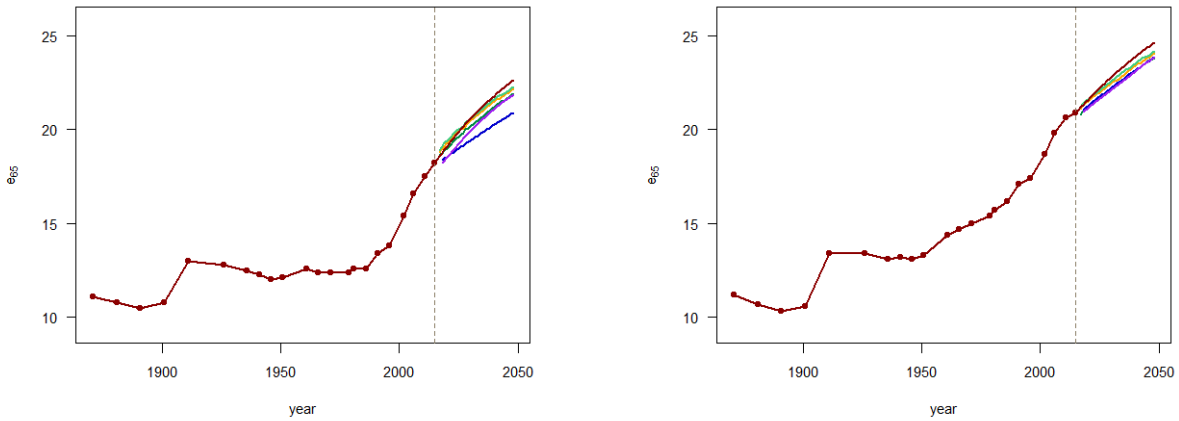
This paper outlines several mortality projection methodologies favoured by official statisticians and academic demographers, and calculated future life expectancies when the different models are applied to Irish data. Table XII summarises some key outputs from these models. It shows that the CSO 2018 projections forecast higher life expectancies than either of the Lee-Carter Model applied to Irish data, or latest UN forecasts for Ireland, and a higher increase in life expectancies than the ONS for Northern Ireland. However, as detailed in earlier subsections treating each methodology, the differences are small in a probabilistic sense – that is, given the large uncertainty inherent in such forecasts, the forecast rates are reasonably close.

Table XII. Observed period life expectancy in Ireland and Northern Ireland at birth and at age 65 by gender projected to 2030, and to 2045.

	Male Life Expectancy		**Female Life Expectancy**		**Gender difference**	
	From Birth	From Age 65 years	From Birth	From Age 65 years	From Birth	From Age 65 years
Observed 2015 Ireland	**79.4**	**18.2**	**83.3**	**20.9**	**3.9**	**2.7**
Projected Values to 2030						
Target Method - CSO	82.6	20.6	85.7	22.8	3.1	2.2
Stochastic Method (Lee-Carter)	81.6	19.4	85.5	22.2	3.9	2.8
Coherent Method (Bayesian) - UN	82.2	19.9	85.2	22.1	3.0	2.2
Projected Values to 2045						
Target Method - CSO	84.8	22.3	87.6	24.4	2.8	2.0
Stochastic Method (Lee-Carter)	83.4	20.7	87.3	23.6	3.9	2.9
Coherent Method (Bayesian) -UN	84.5	21.5	87.0	23.5	2.6	2.0
Observed 2016 Northern Ireland	**78.8**	**18.4**	**82.3**	**20.6**	**3.5**	**2.2**
Projected Values to 2030						
Target Method – ONS for Northern Ireland	81.4	20.2	84.5	22.2	3.1	2.0
Projected Values to 2045						
Target Method – ONS for Northern Ireland	83.3	21.6	86.3	23.6	3.0	2.0

Irish mortality data (like data from other regions) on which the models are calibrated show quite a mixed pattern of changing trends – accelerating and slowing and, at some advanced ages sometimes showing no improvement or even negative trends. Accordingly, the prediction intervals around the above central estimates are wide and widen with each year ahead forecast. It is at ages above age 65 years that most of the uncertainty arises in estimating life expectancies, as changes

to the already very low mortality rates at younger ages have a comparatively minor impact on life expectancy. Figure 10 graphs the expected trajectory of period life expectancies at age 65 years under each of models as calendar years roll on.



(a) e_{65}: Male **(b)** e_{65}: Female

Figure 10. Past and projected evolution of period life expectation at age 65 years, (e_{65}), in Ireland under various models, by gender [—CSO projections, —UK projections, —Northern Ireland projections, —England & Wales projections, —Lee-Carter projections, —UN projections].

There are proposals to link the State pension age with life expectancies from the calendar year 2035 (Government of Ireland (2018)), with a review of the State pension age already planned for the calendar year 2022. A central issue in this review will be how reliably future life expectancies can be estimated. It must not be supposed that the extra mortality data gathered over the next four years or refinements in forecasting techniques in the meantime will help narrow the uncertainty inherent in modelling future mortality. There is more than enough data already on the course of human mortality – from across the regions of the world and across the recent millennia. We ignore most of the past data as it is irrelevant to the future – as today's causes of deaths have changed from the age-old Biblical causes of "by the sword, by famine, by plague, and by the wild animals of the earth". Nor will developments in statistical forecasting technique help much, as the past is only a limited guide to the future, in human mortality as much as in the rest of human destiny.

Mortality rates fell markedly in the past century and longer due to significant improvements in nutrition, housing, public health, education, and medicine. This, in turn, was achieved only by a significant allocation of resources by the individual and the state to achieve this end. Future improvements will require further significant resource allocation and these resources must be directed towards those of older ages (often termed "economically unproductive"). The state plays a significant role in providing income, health care and other services to this subgroup in Ireland, so any changes to such provision can be expected to have an impact on mortality trends.

One suggestion currently mooted is that the future State pension age be set relative to future life expectancy so that the proportion of working life to years in retirement be kept roughly constant, perhaps in the ratio 2:1 (Government of Ireland (2018)). If such a scheme is agreed upon, then it can be construed as a social contract – that the state commits to directing resources to achieving the forecast increases in life expectancies at older ages. Such an understanding would require annual monitoring of mortality improvements against the target rates, and corrective actions in the form

of resource allocations if there is significant deviation. Viewed in such a way, the projections of life expectancies earlier are reasonable targets, believed achievable with a reasonable allocation of resources. With this perspective, the trend in mortality rates in Ireland at ages 90 and over in the last few years would raise an alarm as previous gains in life expectancies are being lost. This also alters the emphasis from mortality forecasting to the more important exercise of monitoring mortality improvements against reasonable targets to help in the allocation of resources. As the British demographer Hajnal remarked

> "... as little forecasting as possible should be done ... Forecasts should flow from analysis of the past. Anyone who has not bothered with analysis should not forecast..."
>
> — Hajnal (1955)

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Appendix A. Life Expectancy Tables

Table XIII. Projected period and cohort life expectancies in 2020 in Ireland on CSO 2018 projection basis, by gender and single year of age.

	Males			Females		
Age	**Period LE in 2020**	**Cohort LE in 2020**	**Gap**	**Period LE in 2020**	**Cohort LE in 2020**	**Gap**
0	80.5	90.4	9.9	84.2	92.7	8.5
1	79.8	89.6	9.8	83.4	91.8	8.4
2	78.8	88.5	9.7	82.4	90.8	8.4
3	77.8	87.4	9.6	81.4	89.7	8.3
4	76.8	86.3	9.5	80.4	88.6	8.2
5	75.8	85.2	9.4	79.4	87.5	8.1
6	74.9	84.1	9.2	78.4	86.4	8.0
7	73.9	83.0	9.1	77.4	85.4	8.0
8	72.9	81.9	9.0	76.4	84.3	7.9
9	71.9	80.8	8.9	75.4	83.2	7.8
10	70.9	79.6	8.7	74.4	82.1	7.7
11	69.9	78.5	8.6	73.4	81.0	7.6
12	68.9	77.4	8.5	72.4	79.9	7.5
13	67.9	76.3	8.4	71.5	78.8	7.3
14	66.9	75.2	8.3	70.5	77.7	7.2
15	65.9	74.1	8.2	69.5	76.6	7.1
16	64.9	73.0	8.1	68.5	75.5	7.0
17	63.9	71.9	8.0	67.5	74.5	7.0
18	62.9	70.8	7.9	66.5	73.4	6.9
19	62.0	69.7	7.7	65.5	72.3	6.8
20	61.0	68.6	7.6	64.5	71.2	6.7
21	60.0	67.5	7.5	63.5	70.1	6.6
22	59.1	66.4	7.3	62.5	69.0	6.5
23	58.1	65.3	7.2	61.5	67.9	6.4
24	57.1	64.2	7.1	60.5	66.8	6.3
25	56.2	63.1	6.9	59.5	65.7	6.2
26	55.2	62.1	6.9	58.6	64.6	6.0
27	54.2	61.0	6.8	57.6	63.5	5.9
28	53.3	59.9	6.6	56.6	62.5	5.9
29	52.3	58.8	6.5	55.6	61.4	5.8
30	51.3	57.7	6.4	54.6	60.3	5.7
31	50.4	56.6	6.2	53.6	59.2	5.6
32	49.4	55.5	6.1	52.7	58.1	5.4
33	48.4	54.4	6.0	51.7	57.0	5.3
34	47.5	53.3	5.8	50.7	55.9	5.2
35	46.5	52.3	5.8	49.7	54.8	5.1
36	45.5	51.2	5.7	48.7	53.7	5.0
37	44.6	50.1	5.5	47.7	52.7	5.0
38	43.6	49.0	5.4	46.8	51.6	4.8
39	42.7	47.9	5.2	45.8	50.5	4.7
40	41.7	46.8	5.1	44.8	49.4	4.6
41	40.7	45.7	5.0	43.8	48.3	4.5
42	39.8	44.6	4.8	42.9	47.2	4.3
43	38.8	43.5	4.7	41.9	46.1	4.2
44	37.9	42.5	4.6	40.9	45.1	4.2
45	36.9	41.4	4.5	40.0	44.0	4.0
46	36.0	40.3	4.3	39.0	42.9	3.9
47	35.0	39.2	4.2	38.0	41.8	3.8

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	Males			Females		
Age	**Period LE in 2020**	**Cohort LE in 2020**	**Gap**	**Period LE in 2020**	**Cohort LE in 2020**	**Gap**
48	34.1	38.2	4.1	37.1	40.8	3.7
49	33.2	37.1	3.9	36.1	39.7	3.6
50	32.2	36.1	3.9	35.2	38.6	3.4
51	31.3	35.0	3.7	34.2	37.6	3.4
52	30.4	34.0	3.6	33.3	36.5	3.2
53	29.5	32.9	3.4	32.4	35.5	3.1
54	28.6	31.9	3.3	31.4	34.4	3.0
55	27.7	30.9	3.2	30.5	33.4	2.9
56	26.8	29.9	3.1	29.6	32.4	2.8
57	25.9	28.9	3.0	28.7	31.4	2.7
58	25.0	27.9	2.9	27.8	30.4	2.6
59	24.2	26.9	2.7	26.9	29.4	2.5
60	23.3	25.9	2.6	26.0	28.4	2.4
61	22.4	24.9	2.5	25.1	27.4	2.3
62	21.6	23.9	2.3	24.2	26.4	2.2
63	20.8	23.0	2.2	23.3	25.4	2.1
64	19.9	22.0	2.1	22.5	24.4	1.9
65	19.1	21.1	2.0	21.6	23.4	1.8
66	18.3	20.2	1.9	20.7	22.5	1.8
67	17.5	19.3	1.8	19.9	21.5	1.6
68	16.7	18.4	1.7	19.0	20.6	1.6
69	15.9	17.5	1.6	18.2	19.7	1.5
70	15.2	16.6	1.4	17.4	18.8	1.4
71	14.4	15.8	1.4	16.6	17.9	1.3
72	13.7	15.0	1.3	15.8	17.0	1.2
73	13.0	14.2	1.2	15.0	16.1	1.1
74	12.3	13.4	1.1	14.2	15.3	1.1
75	11.6	12.6	1.0	13.5	14.4	0.9
76	10.9	11.8	0.9	12.7	13.6	0.9
77	10.3	11.1	0.8	12.0	12.8	0.8
78	9.6	10.4	0.8	11.3	12.0	0.7
79	9.0	9.7	0.7	10.6	11.2	0.6
80	8.4	9.1	0.7	9.9	10.5	0.6
81	7.9	8.4	0.5	9.3	9.8	0.5
82	7.3	7.8	0.5	8.6	9.1	0.5
83	6.8	7.2	0.4	8.0	8.4	0.4
84	6.3	6.6	0.3	7.5	7.8	0.3
85	5.8	6.1	0.3	6.9	7.2	0.3
86	5.4	5.7	0.3	6.4	6.7	0.3
87	5.0	5.2	0.2	5.9	6.1	0.2
88	4.6	4.8	0.2	5.5	5.7	0.2
89	4.3	4.4	0.1	5.1	5.2	0.1
90	4.0	4.1	0.1	4.7	4.8	0.1
91	3.6	3.7	0.1	4.3	4.4	0.1
92	3.4	3.4	0.0	3.9	4.0	0.1
93	3.1	3.1	0.0	3.6	3.7	0.1
94	2.8	2.9	0.1	3.3	3.3	0.0
95	2.6	2.6	0.0	3.0	3.1	0.1
96	2.4	2.4	0.0	2.8	2.8	0.0
97	2.3	2.3	0.0	2.6	2.6	0.0
98	2.1	2.1	0.0	2.4	2.4	0.0
99	2.0	2.0	0.0	2.2	2.2	0.0
100	1.8	1.8	0.0	2.1	2.1	0.0