Disclaimer

The views expressed in this presentation are those of the presenter(s) and not necessarily those of the Society of Actuaries in Ireland or their employers.
Motivation

“I was just guessing numbers and figures”
Chris Martin,
The Scientist
Key Ideas

1. Parameter Uncertainty matters...
2. ...and it matters more for reinsurance
3. Stop using excel!
Who am I?

- Chris Gibney
- Pricing/Capital Actuary
- Lloyds
- Insurance/Reinsurance/Retrocession
- Most lines of business
Reinsurance

- Who works in reinsurance on a day to day basis?
- What is excess of loss reinsurance?
Reinsurance

Excess of Loss Reinsurance

EUR 5m, FGU

EUR 7m, Limit

EUR 2m, Recovery

EUR 3m, Excess

FGU Claim

Excess of Loss
Reinsurance

- Reinsurance is non linear
- Gearing effect
Reinsurance

Excess of Loss Reinsurance

EUR million

FGU Claim

Excess of Loss

2m
Reinsurance

Excess of Loss Reinsurance

FGU +1m (+20%)
RI Loss +1m (+50%)
Gearing Effect

![Graph showing the Gearing Effect with lines labeled 1x1, 3x2, and FGU.]
Gearing Effect

![Graph showing the gearing effect with lines for 3x2, 1x1, and FGU, indicating how implied inflation changes with FGU inflation.](image)
Gearing Effect

Graph showing the relationship between FGU Inflation and Implied Inflation for different gearing effects (1x1, 3x2). The graph illustrates how the implied inflation increases with FGU inflation for different gearing ratios.
Gearing Effect

Graph made in R
Code on SAI website
Parameter Error

- It's important to get your parameters right...
- ...Small error in parameters = big error in RI loss cost!
Bayes vs Frequentist

- $X = \text{Claim Severity}$
- **Frequentist**
  - $X \sim \text{LogN}(\mu, \sigma)$
- **Bayes**
  - $X | \mu, \sigma \sim \text{LogN}(\mu, \sigma)$
  - $\mu \sim ???$
  - $\sigma \sim ???$
Bayes vs Frequentist
Bayes Formula

$$P(\text{Parameter} \mid \text{Data}) = \frac{P(\text{Data} \mid \text{Parameter}) \times P(\text{Parameter})}{P(\text{Data})}$$
Bayes Formula

- \( P(\text{Parameter} \mid \text{Data}) = \frac{P(\text{Data} \mid \text{Parameter}) \times P(\text{Parameter})}{P(\text{Data})} \)

- \( P(\text{Parameter} \mid \text{Data}) \propto P(\text{Data} \mid \text{Parameter}) \times P(\text{Parameter}) \)
Markov Chain Monte Carlo

- **Markov Chain**
  - Stochastic Process
  - What happens next only depends on the current state
  - Equilibrium distribution (Ergodic Theorem)
Markov Chain
Monte Carlo

► Monte Carlo
  ► A luxury holiday spot in southern France
  ► Or a lab at Los Alamos in California

► Calculate expected values using simulation
Metropolis Algorithm

- $X =$ Claim Severity
- Model:
  - $X \mid \mu \sim \log N(\mu, \sigma)$
  - $\mu \sim f(\theta)$
- We want to find $P(\mu \mid X)$
- So how does the algorithm work?
Metropolis Algorithm

English:
1. Start with some initial value
2. Propose an alternative
3. Determine which is a better fit
4. If the proposal is a better fit, accept it
5. If the proposal is a better fit, accept it with a certain probability
Metropolis Algorithm

Maths:

1. Start with an initial value $\mu_{current}$
2. Propose an alternative, $\mu_{proposal}$
3. Figure out which is a better fit:
   - $probability_{current} \propto P(X | \mu_{current}, \sigma) \times P(\mu_{current} | \theta)$
   - $probability_{proposal} \propto P(X | \mu_{proposal}, \sigma) \times P(\mu_{proposal} | \theta)$
4. IF $probability_{proposal} > probability_{current}$
   - $\mu_{current} = \mu_{proposal}$
5. ELSE
   - Set Acceptance Probability = $\frac{probability_{proposal}}{probability_{current}}$
Metropolis Algorithm

- How to come up with a proposal?
- $\mu_{\text{proposal}} \sim N(\mu_{\text{current}}, s)$
- $s = \text{proposal width}$, algorithm parameter
- Markov Chain
- Symmetric distribution
- Ergodic theorem
Metropolis Algorithm

- In R
Prior

- It’s a spectrum
- $\text{probability}_{\text{proposal}} \propto P(X \mid \mu_{\text{proposal}}, \sigma) \times P(\mu_{\text{proposal}} \mid \theta)$
Burn in

![Graphs showing burn-in periods with y-axes labeled results$current[1:5000] and results$current[1:50000].]
Proposal Width

Proposal Width 0.5

Proposal Width 5.0
Auto Correlation

Using all simulations

Using every 20\textsuperscript{th} simulation

Efficient?
Conclusion

- MCMC is... a tool
  - I like it coz I like Bayes
  - I like it coz its easy to implement
  - I like it coz it lets me incorporate benchmark/prior
- ...but MCMC is just a tool
Conclusion

- We have just scratched the surface...
- ...if you like it, let the SAI know!
- ...and if you didn’t like it ...
- Code on website