Society of Actuaries in Ireland

Intro to Markov Chains Monte Carlo and Excess of Loss Pricing

Thursday 27 June 2019

Disclaimer

The views expressed in this presentation are those of the presenter(s) and not necessarily those of the Society of Actuaries in Ireland or their employers.

Motivation

"I was just guessing numbers and figures"

Chris Martin,

The Scientist

Key Ideas

1. Parameter Uncertainty matters…
2. …and it matters more for reinsurance
3. Stop using excel!

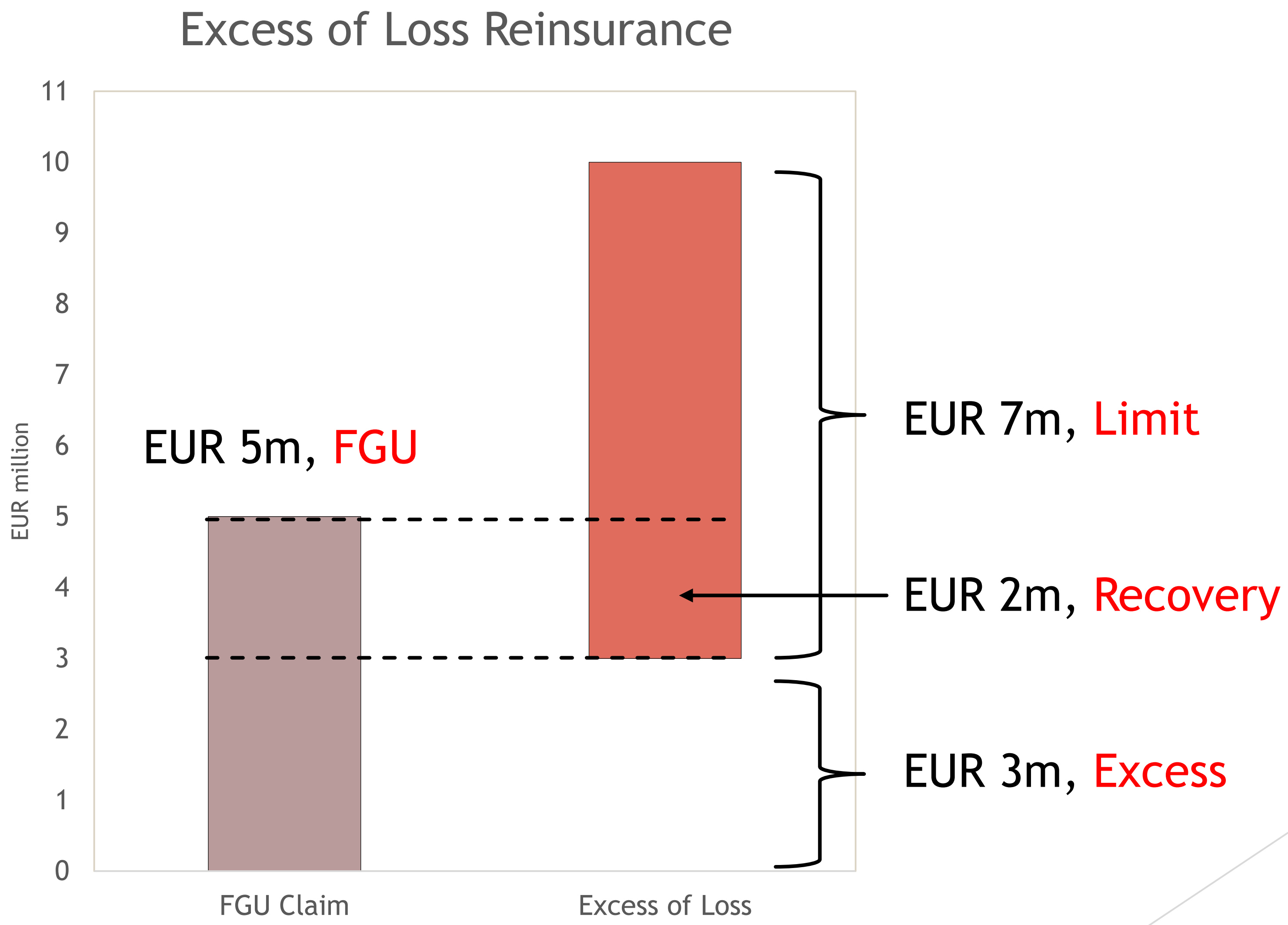
Who am I?

- Chris Gibney
- Pricing/Capital Actuary
- Lloyds
- Insurance/Reinsurance/ Retrocession
- Most lines of business

Reinsurance

- Who works in reinsurance on a day to day basis?
- What is excess of loss reinsurance?

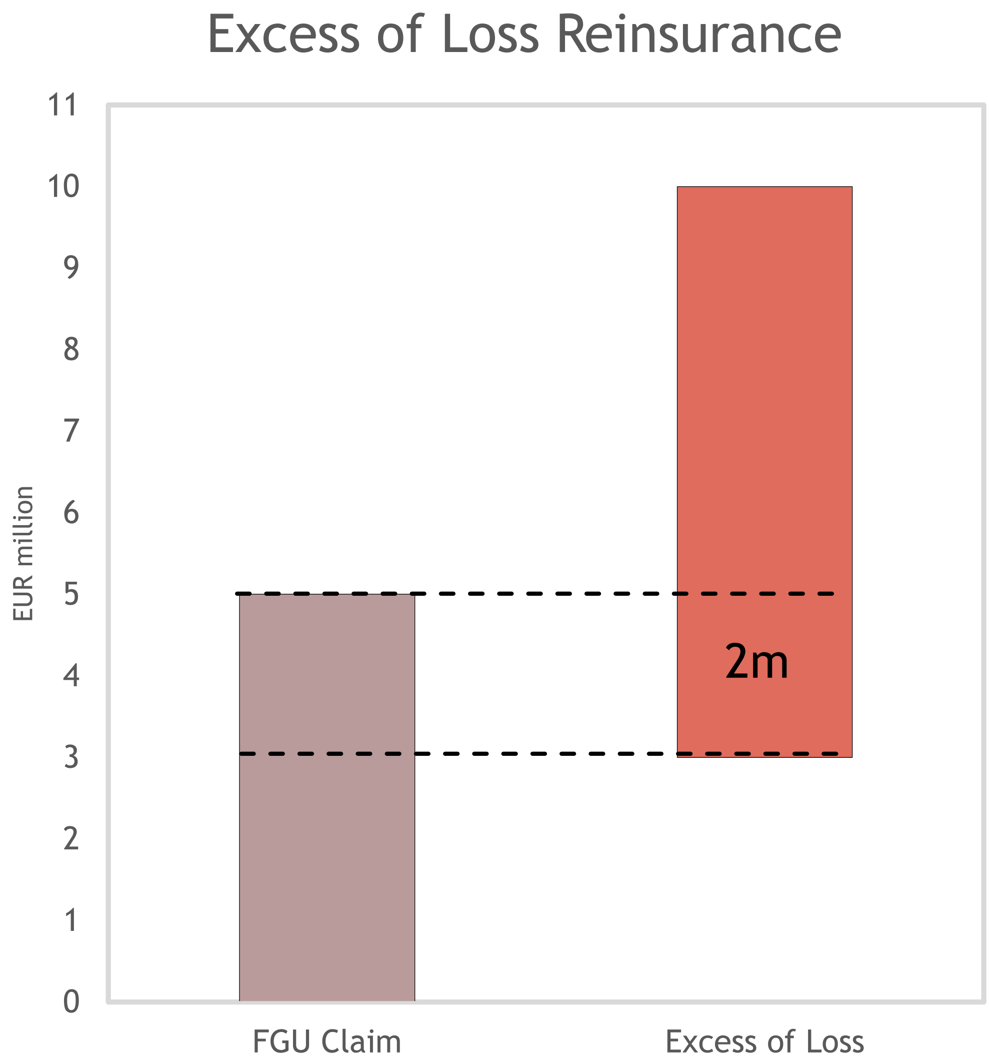
Reinsurance



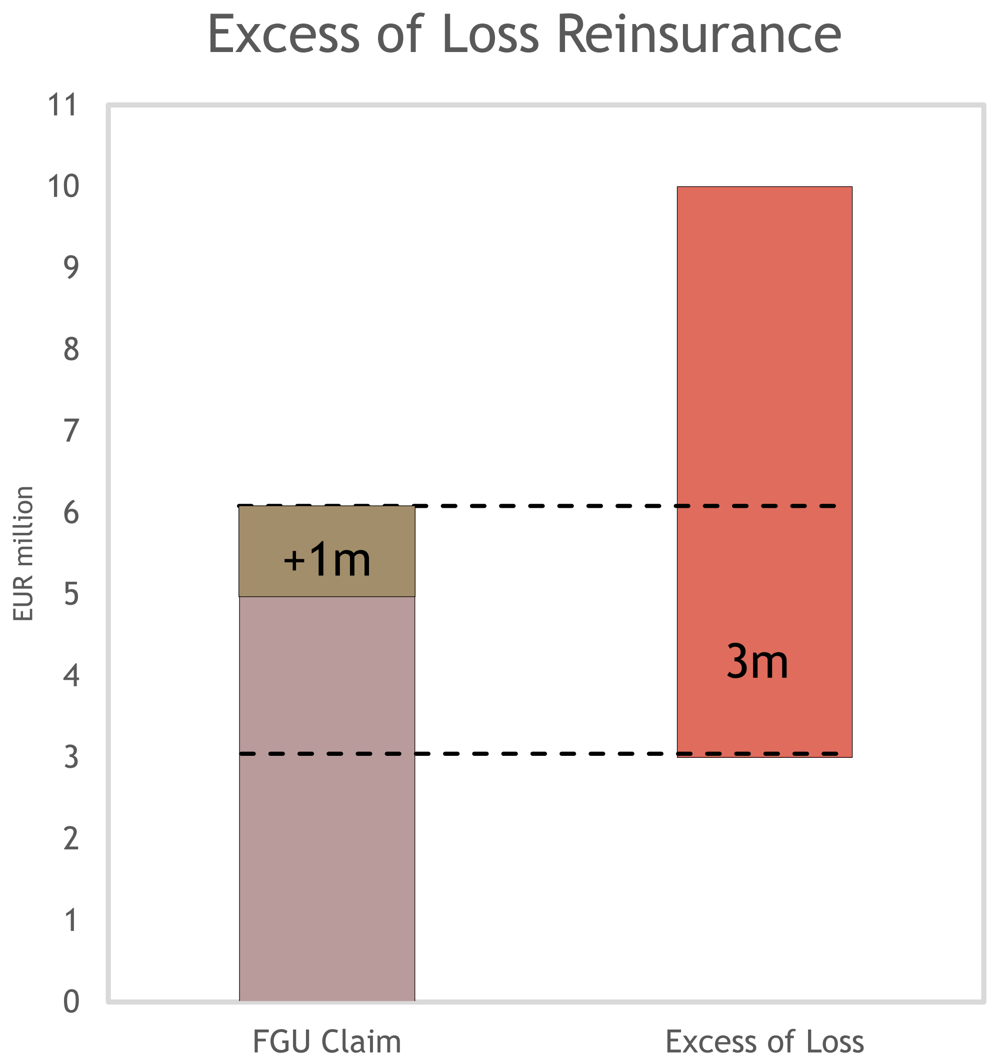
Reinsurance

- Reinsurance is non linear
- Gearing effect

Reinsurance



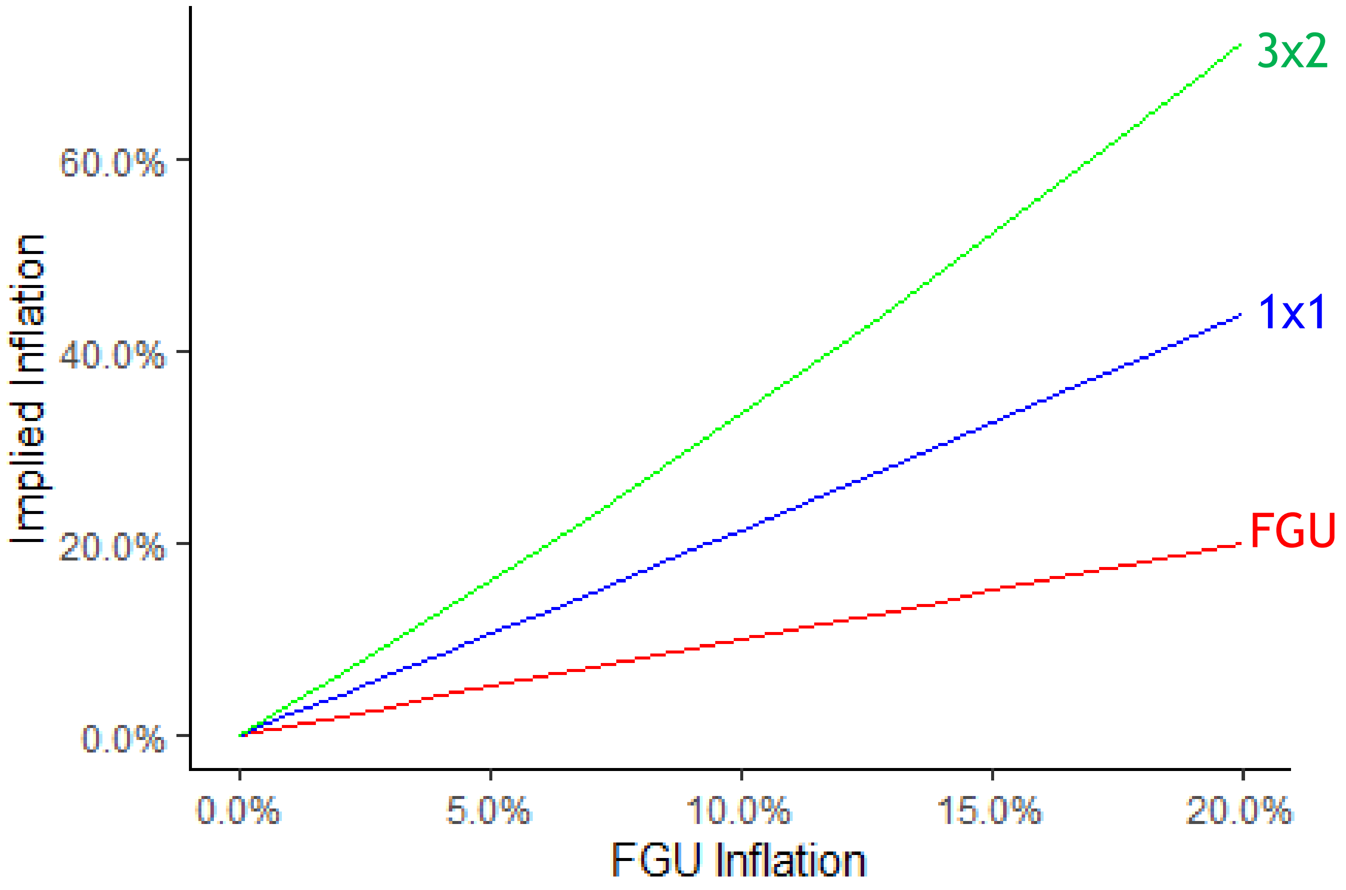
Reinsurance



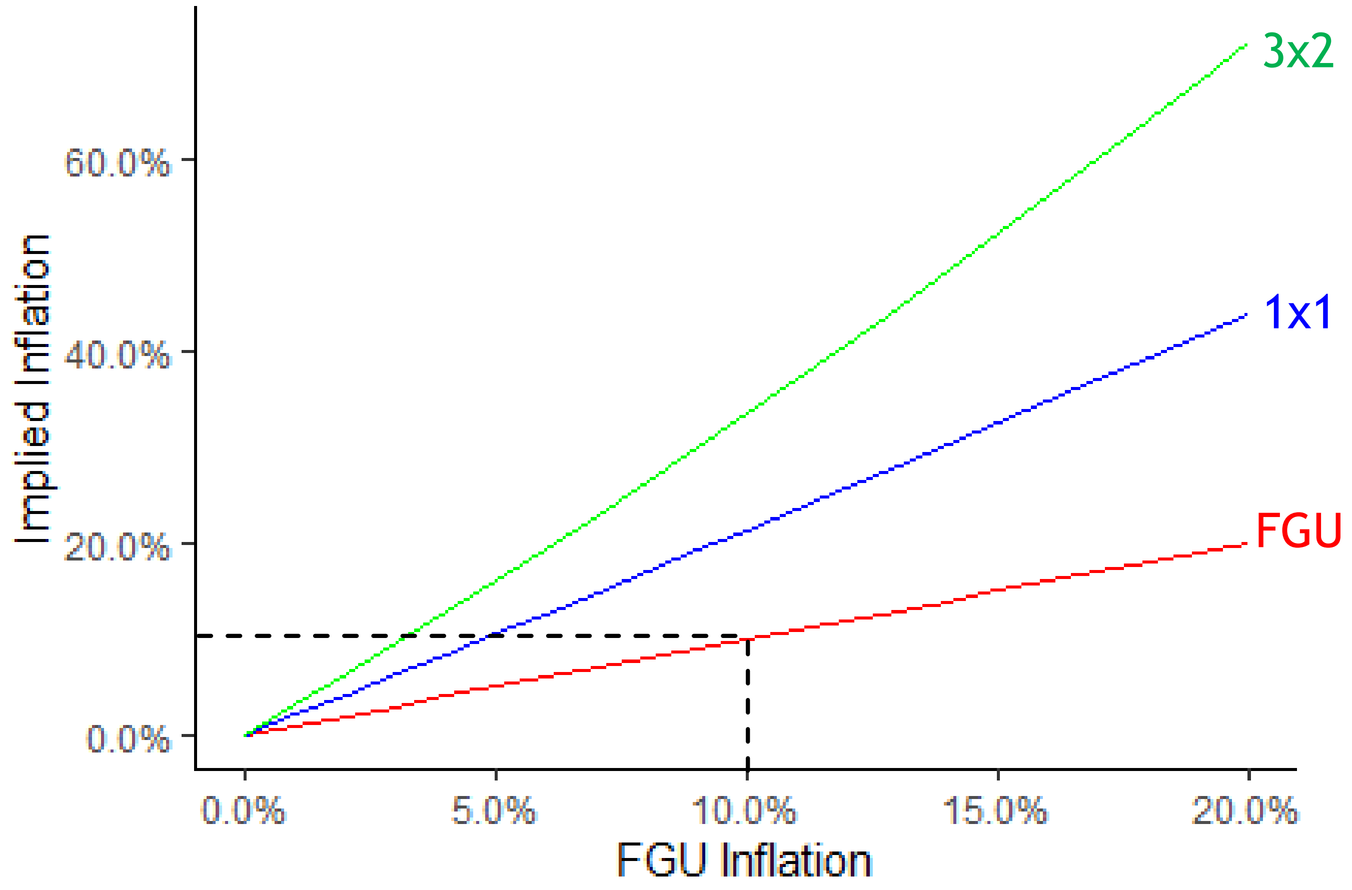
FGU +1m (+20%)

RI Loss +1m (+50%)

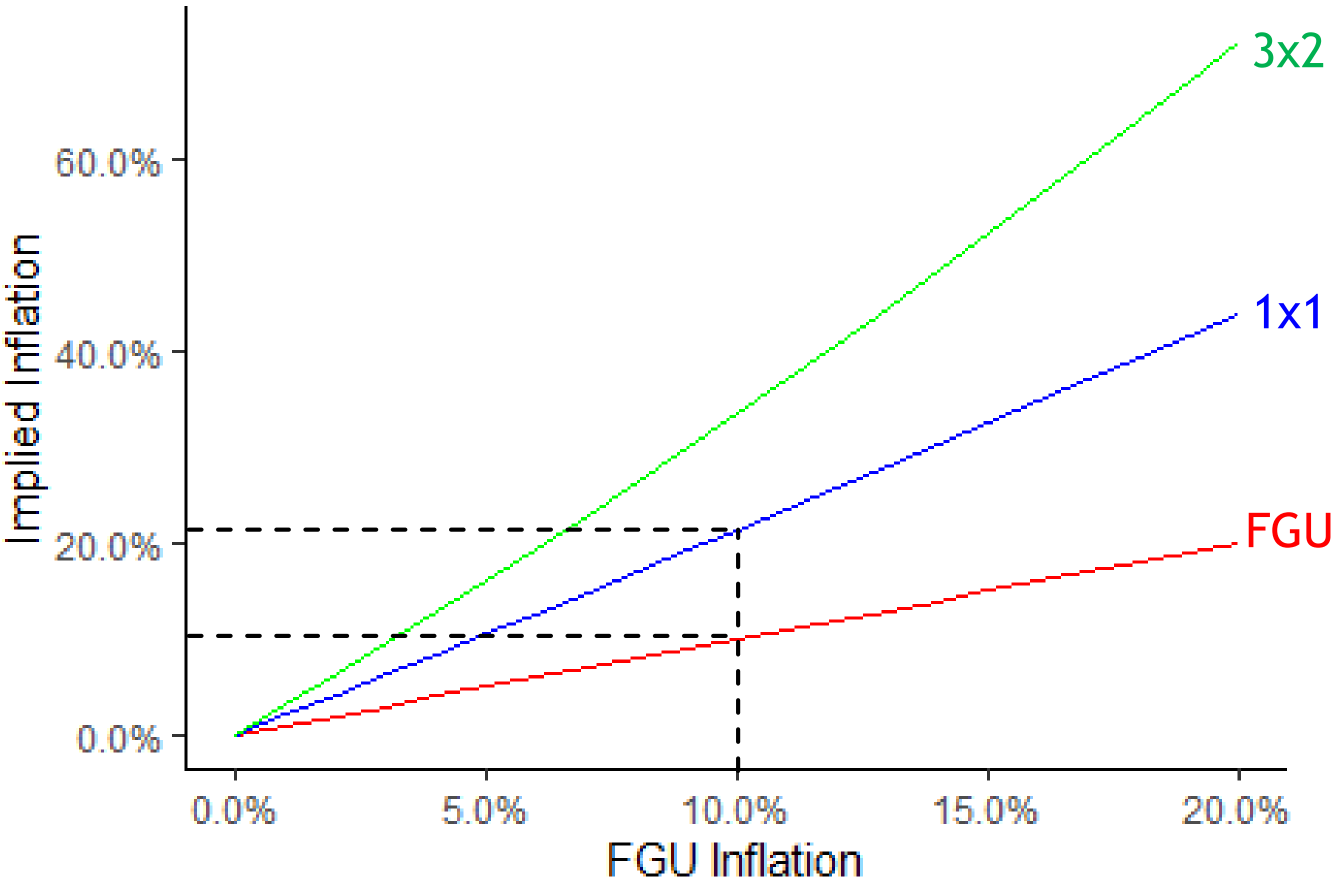
Gearing Effect



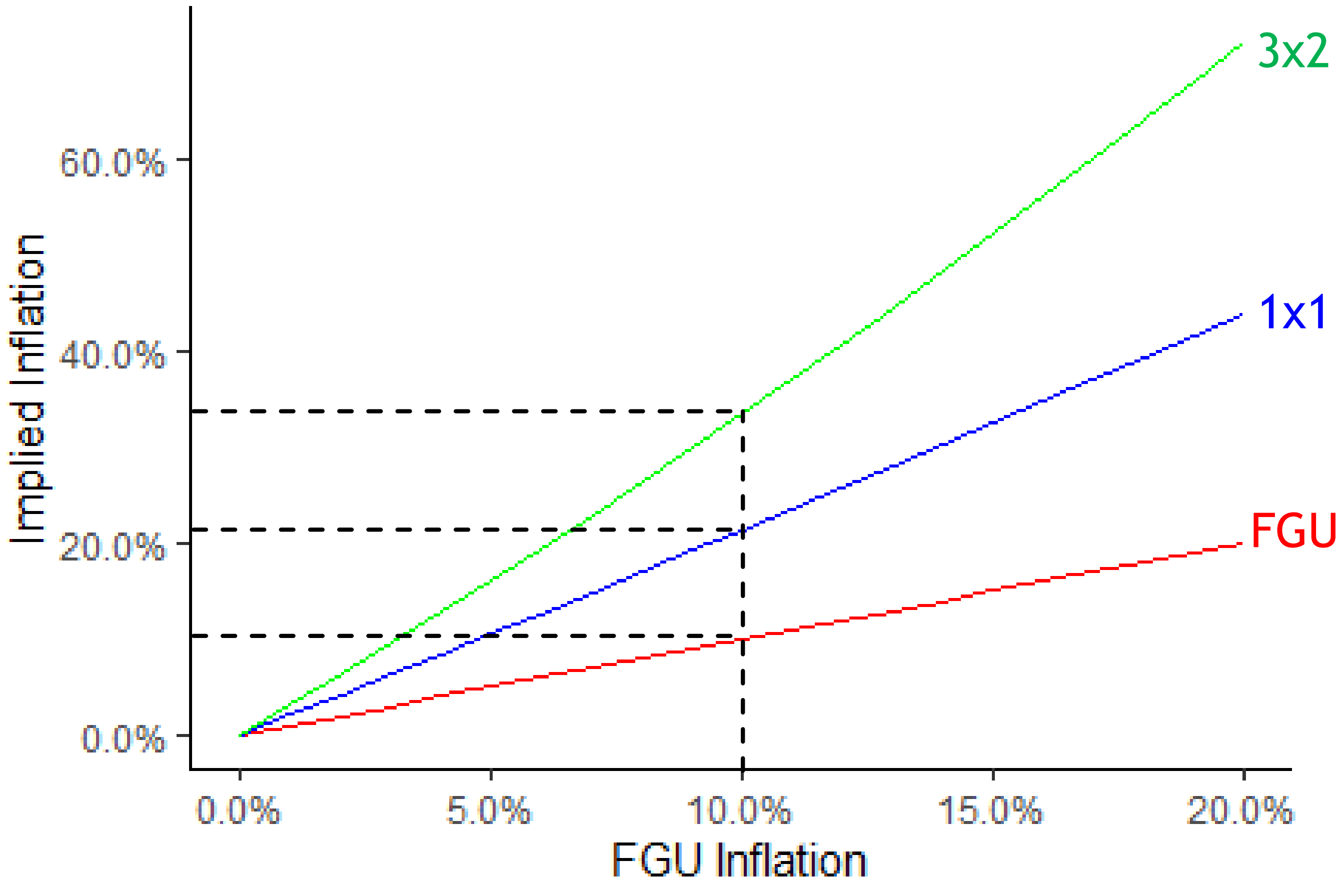
Gearing Effect



Gearing Effect



Gearing Effect



Graph made in R

Code on SAI website

Parameter Error

- Its important to get your parameters *right*…
- …Small error in parameters = big error in RI loss cost!

Bayes vs Frequentist

- X = Claim Severity
- Frequentist
  - $X \sim LogN(\mu, \sigma)$
- Bayes
  - $X \mid \mu, \sigma \sim LogN(\mu, \sigma)$
  - $\mu \sim ???$
  - $\sigma \sim ???$

Bayes vs Frequentist

Bayes Formula

Likelihood

Prior

- $$P(Parameter \mid Data) = \frac{P(Data \mid Parameter) \times P(Parameter)}{P(Data)}$$

Posterier

Evidence

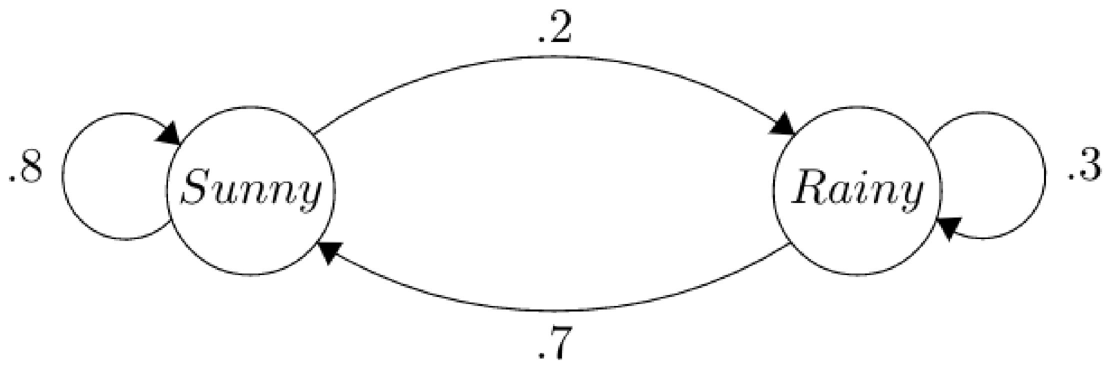
Bayes Formula

- $P(Parameter \mid Data) = \frac{P(Data \mid Parameter) \times P(Parameter)}{P(Data)}$

- $P(Parameter \mid Data) \propto P(Data \mid Parameter) \times P(Parameter)$

Markov Chain Monte Carlo

- Markov Chain
  - Stochastic Process
  - What happens next only depends on the current state
  - Equilibrium distribution (Ergodic Theorem)



Markov Chain Monte Carlo

- Monte Carlo
  - A luxury holiday spot in southern France
  - Or a lab at Los Alamos in California
- Calculate expected values using simulation

Metropolis Algorithm

- X = Claim Severity
- Model:
  - $X \mid \mu \sim LogN(\mu, \sigma)$
  - $\mu \sim f(\theta)$
- We want to find $\mathrm{P}(\mu \mid X)$
- So how does the algorithm work?

Metropolis Algorithm

- English:
  1. Start with some initial value
  2. Propose an alternative
  3. Determine which is a better fit
  4. If the proposal is a better fit, accept it
  5. If the proposal is a better fit, accept it with a certain probability

Metropolis Algorithm

Maths:

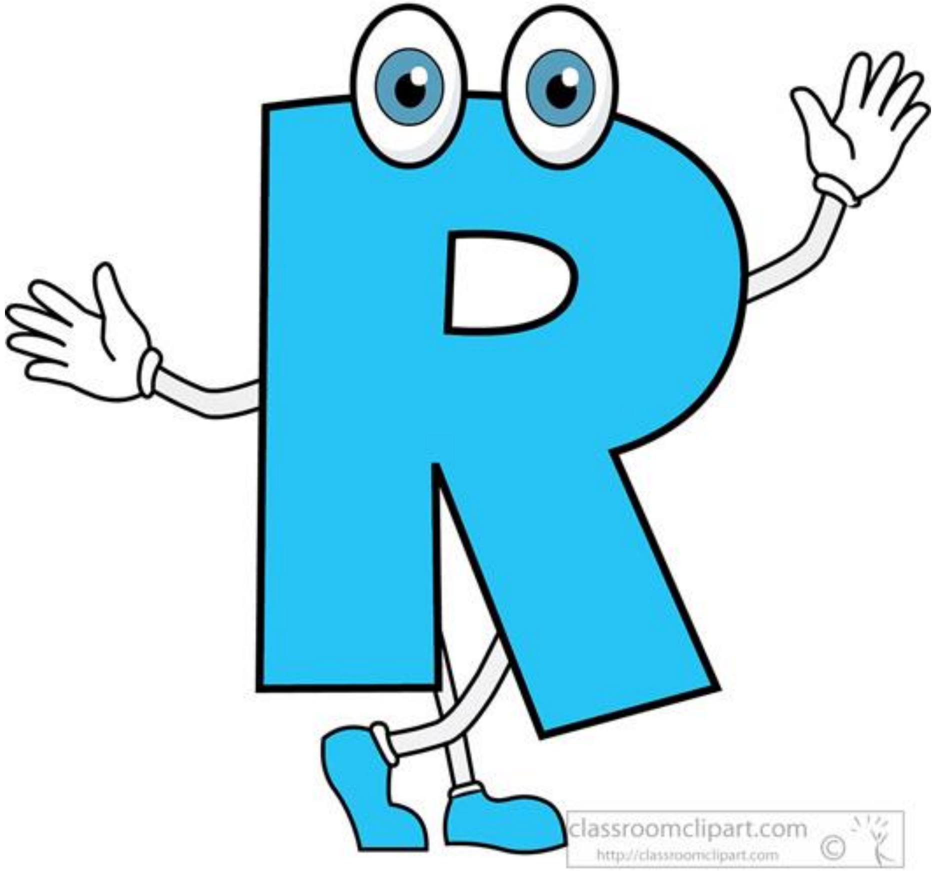
1. Start with an initial value $\mu_{current}$
2. Propose an alternative, $\mu_{proposal}$
3. Figure out which is a better fit:
   - $probability_{current} \propto P(X \mid \mu_{current}, \sigma) \times P(\mu_{current} \mid \theta)$
   - $probability_{proposal} \propto P(X \mid \mu_{proposal}, \sigma) \times P(\mu_{proposal} \mid \theta)$
4. IF $probability_{proposal} > probability_{current}$
   - $\mu_{current} = \mu_{proposal}$
5. ELSE
   - Set Acceptance Probability $= \frac{probability_{proposal}}{probability_{current}}$

Metropolis Algorithm

- How to come up with a proposal?
- $\mu_{proposal} \sim N(\mu_{current}, s)$
- s = *proposal width*, algorithm parameter
- Markov Chain
- Symmetric distribution
- Ergodic theorum

Metropolis Algorithm

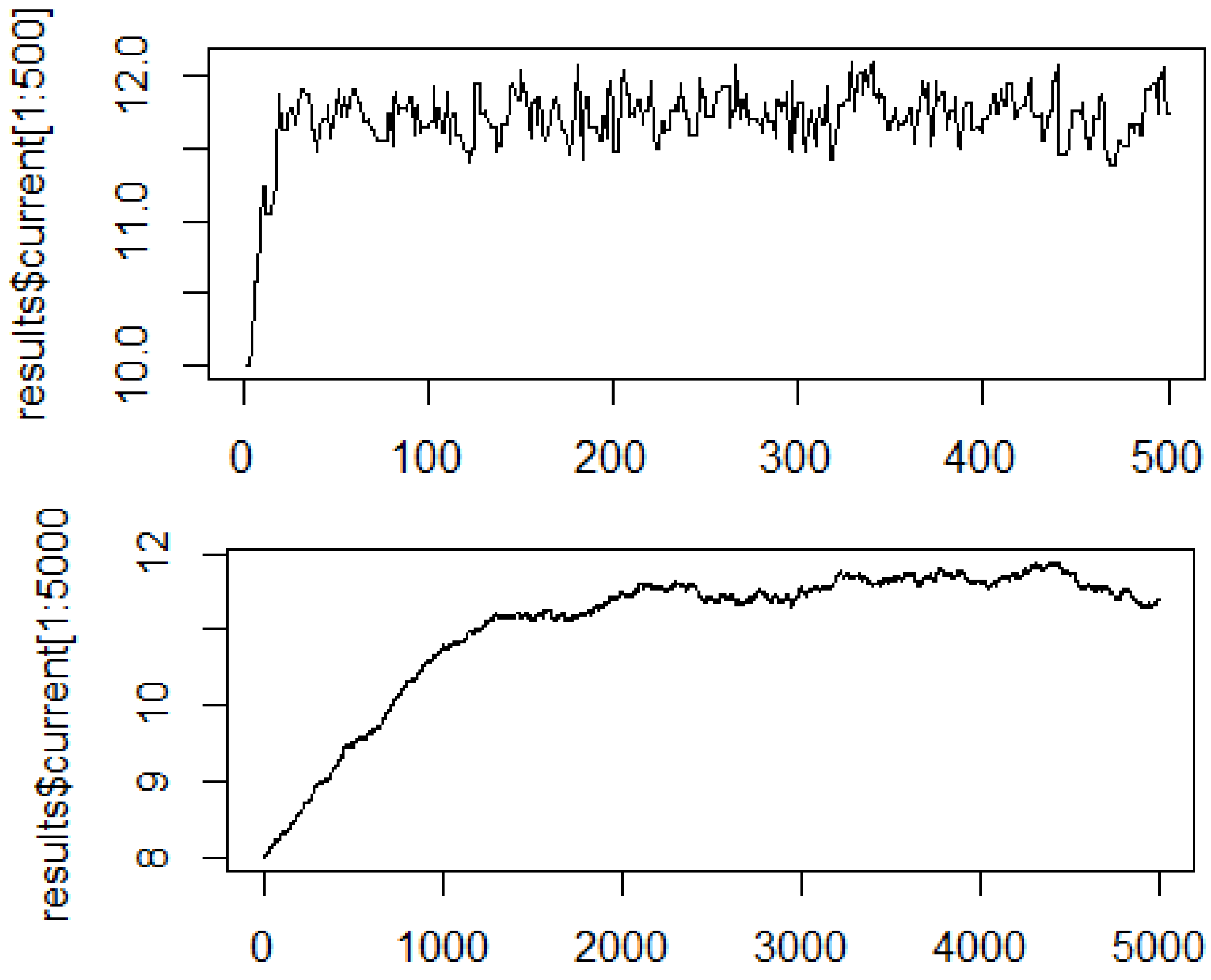
- In R



Prior

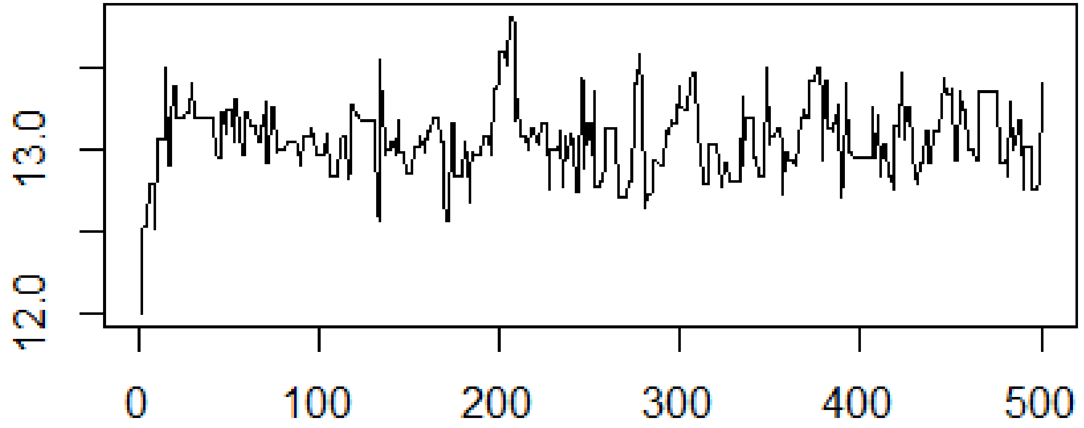
- It's a spectrum
- $probability_{proposal} \propto P(X \mid \mu_{proposal}, \sigma) \times P(\mu_{proposal} \mid \theta)$

Burn in

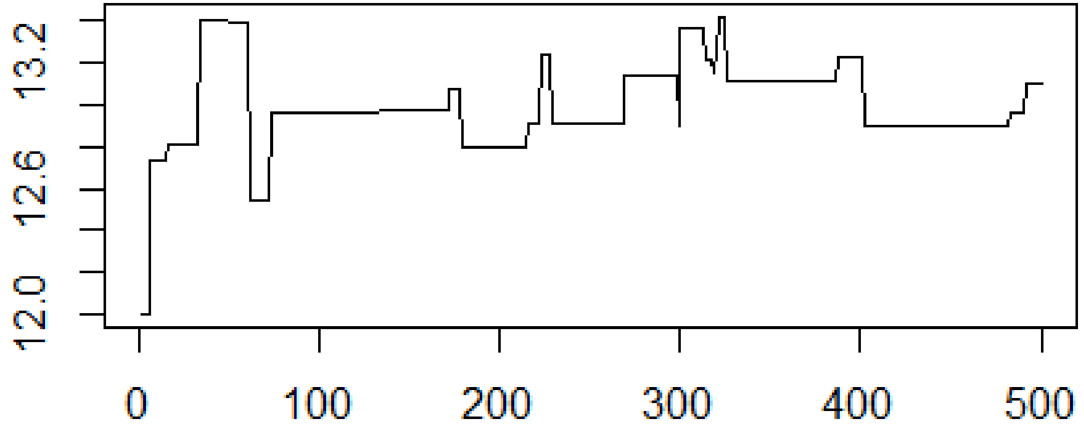


results$current[1:500]
10.0
11.0
12.0
0
100
200
300
400
500
results$current[1:5000]
8
9
10
12
0
1000
2000
3000
4000
5000

Proposal Width

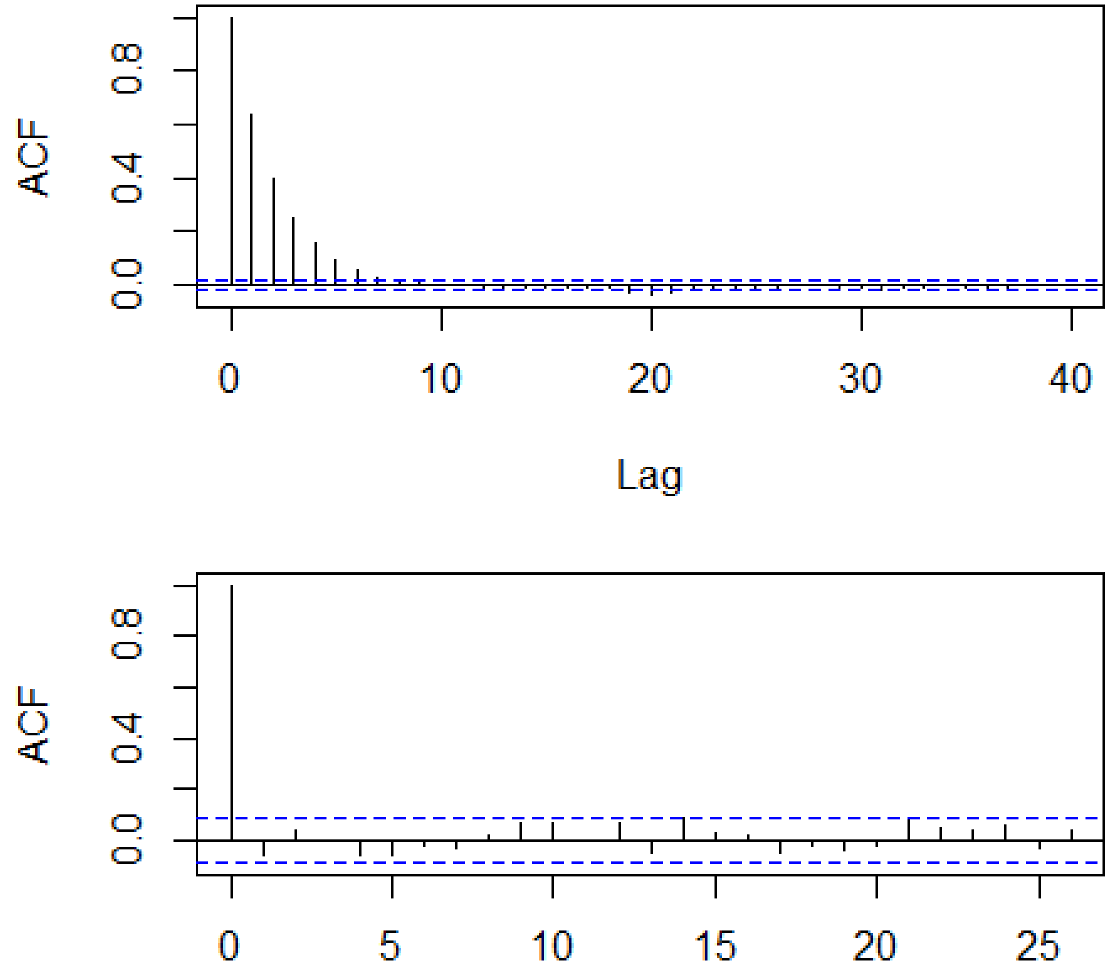


Proposal Width 0.5



Proposal Width 5.0

Auto Correlation



Using all simulations

Using every 20th simulation

Efficient?

Conclusion

- MCMC is… a tool
  - I like it coz I like Bayes
  - I like it coz its easy to implement
  - I like it coz it lets me incorporate benchmark/prior
- …but MCMC is just a tool

Conclusion

- We have just scratched the surface…
- …if you like it, let the SAI know!
- …and if you didn’t like it …
- Code on website

Questions