Society of Actuaries in Ireland

A Ramble Though Some Historical Actuarial Countryside

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Topics for Today

- Illiquid asset investing by British life offices in the late 19th century
- The actuarial development of market risk management techniques in the 1950s
- Risk-based capital and long-term financial guarantees in the late 20th century

Illiquid Investing by British Life Offices in the late 19th century

Background on actuarial thought and practice in investment / ALM in mid-19th century

- Little thinking published by actuaries on investment / ALM in mid-19th century
- But an increasing sense that mortality modelling was mastered, and profits from that source would be limited
- Mid-19th century life office asset allocation ~ 50% gilts / 50% mortgages
- Long-term interest rates had fallen for several decades since end of Napoleonic Wars
  - Becoming a challenge to earn with-profit premium rate of 3%

British Life Office Asset Allocation in 1890s

An Appetite for Illiquid Assets

- British life offices invested substantially in non-exchange traded, illiquid asset classes in the late 19th century

- This tendency peaked in the 1890s, when the average British life office allocated **80%** of assets to non-exchange traded asset classes:
  - 52% mortgages;
  - 6% property;
  - 22% loans.

- The remainder was mainly held in cash and government bonds (colonial as well as gilts)
- Mackenzie (1891)

Bailey and Illiquid Asset Investing (1862)

- British actuarial thought in mid-19th century life assurance was predominantly focused on mortality modelling and with-profit reserving methods
- A few papers were published on investment strategy; most notable was A.H. Bailey's 1862 paper (Bailey was Institute President 1878-1882)
- Bailey on life assurers' uniquely illiquid form of long-term liabilities:

> "The probable amount of demands on their [life assurers'] resources can be calculated from time to time within not very wide limits.
>
> Unlike banks…they are not exposed to sudden or unusual demands on their resources in times of panic."

- Bailey on the presence of an asset illiquidity premium:

> "The much larger proportion [of life office assets] may safely be invested in securities that are ***not readily convertible***; and it is desirable…that it should be so invested, because such securities, being unsuited for private individuals and trustees, ***command a higher rate of interest in consequence***"

The Major Rotation from Illiquid to Liquid Assets in early 20th century

- By 1920, the average British life office allocation to illiquid assets had fallen to less than 40%
  - Mortgage investments reduced from 52% to 15% between 1890 and 1920
- Why this great rotation from illiquid to liquid assets?
  - Falls in agricultural land values (mortgages mainly written on farms)
  - Increase in lapse rates and recognition that liabilities were not as illiquid as previously assumed
  - Patriotic buying of government bonds during First World War
- This trend continued through the 20th century; post-second World War II life office asset allocation changes dominated by increasing allocations to equities
- See Dodds (1979)

The actuarial development of market risk management techniques in the 1950s

Actuaries and interest rates in early 20th century

- No significant formal actuarial research on interest rate risk prior to WW II
- Perhaps because asset / liability valuations were not mark-to-market
  - Assets at lower of book and market values
  - Liabilities based on assumed long-term interest rate
- Further, with-profit business did not necessarily require interest rate hedging
  - e.g. future regular premiums expected to participate in any increase in interest rates
  - The major actuarial controversy that followed Redington's paper was about how to apply it to with-profit liabilities (paid up; accrued guarantee; future expected bonuses)

Macaulay duration (1938)

- Frederick Macaulay was a Canadian academic economist
  - Son of T.B. Macaulay, Actuary of Sun Life of Canada (and Scots immigrant)

- His measure of duration as present value-weighted average term of bond cashflows was published in 1938 in empirical study of US government bond yield curves
- Was an *ad-hoc* measure of empirical bond price sensitivity to interest rates
- No recognition that it was first derivative of bond price equation or its risk management possibilities
- Features in 3 pages of a 600 page tome on historical government bond prices
- Un-noted by bond managers and actuaries for decades

An Actuarial Anticipation of Immunization (1937)

- Redington's immunization theory was published in 1952; biggest contribution actuarial science made to wider financial risk management theory and practice
- But conceptual idea was in the ether of British actuarial thought decades before Redington's mathematical codification.

- In a 1937 Faculty sessional meeting on life investment strategy, J.D. Binns (Investment Secretary of Scottish Widows) stated:

> "Owing to the incidence of future income, it will be found that assets should be made to mature at rather later dates than the corresponding liabilities.
>
> Then should interest rates fall, the future income will be accumulated at a lower rate of interest, but this would be compensated by the appreciation shown by the assets at the time that the corresponding liabilities fall due (and vice versa)."

- This is a statement of what an immunized portfolio *does*, but does not include a process for *identifying or constructing the portfolio*

Redington's Immunisation Theory (1952)

> "'…if [asset and liability] present values were equal at one rate of interest and remained equal on a shift in the rate of interest this is only the layman's way of saying that the differentials of the present value with regard to the rate of interest must be equal."

- Assumes flat yield curve with interest rate i:
- $v = 1 / (1+i)$
- $V_A = \sum v^t A_t; V_L = \sum v^t L_t$

- First immunization condition: $\frac{dV_A}{di} = \frac{dV_L}{di}; \sum t v^t A_t = \sum t v^t L_t$
- Second immunization condition: $\frac{d^2 V_A}{di^2} > \frac{d^2 V_L}{di^2}; \sum t^2 v^t A_t > \sum t^2 v^t L_t$

Issues and Limitations with Immunization – Haynes and Kirton (1952)

- Flat yield curve that only moves in parallel produces arbitrage
  - Redington's immunisation portfolio is generating arbitrage profits
- Ignores risk that yield curve does not move in parallel
- Assumes, for example, a 10-year cashflow can be perfectly hedged by a smaller exposure to a 30-year cashflow
- Optimal immunizing portfolio is mix of cash and the longest-duration asset
- This maximises exposure to the risk ignored by the model
- These conclusions and limitations were noted in Haynes and Kirton (1952) paper - written contemporaneously and without knowledge of Redington paper:

> "'… any such combination of dead-short and dead-long investments offers no protection against a change in interest rates which is not uniform for all terms."

Risk management by dynamic asset allocation: Background of increasing equity investment

- UK with-profit funds increased their allocation to equities from 2% to 21% between 1920 and 1952

- This occurred in the broader context of ‘the cult of equities’ that was prevalent amongst long-term institutional investors
  - Inflation shock of First World War => desire for real assets; recognition that long nominal bonds could be very volatile, challenging solvency (especially if liabilities valued at static interest rate)
  - Keynes, Raynes highlighting strong long-term equity performance

- Virtually no professional research or discussion on equity risk or what allocation to equities should be targeted by with-profit funds
  - Large with-profit estates
  - Positive yield gap (dividend yield > gilt yield)
  - Equities valued at lesser of book value and market value
  - Actuarial discussion focused mainly on bonus policy in context of equities, rather than the solvency risks created by equity allocation

Early consideration of with-profit liabilities in equity allocation

- In sessional meeting discussion of Raynes' equity paper of 1938, Recknell, Actuary at National Mutual (Keynes' office) commented:

> "Concerning the percentage of ordinary shares which should be purchased there was something to be said, as a very rough rule, for investing on more or less conventional lines to the extent of the funds needed to support contractual liabilities, while leaving the surplus free to invest in ordinary shares and real estate."

- Intuitive rule, though gross premium valuation could imply 100% equities allocation for new policies

Anderson and Binns (1957)

- Anderson and Binns proposed investing more in equities than implied by Recknell, by recognising that equities were very unlikely to fall to a value of zero:

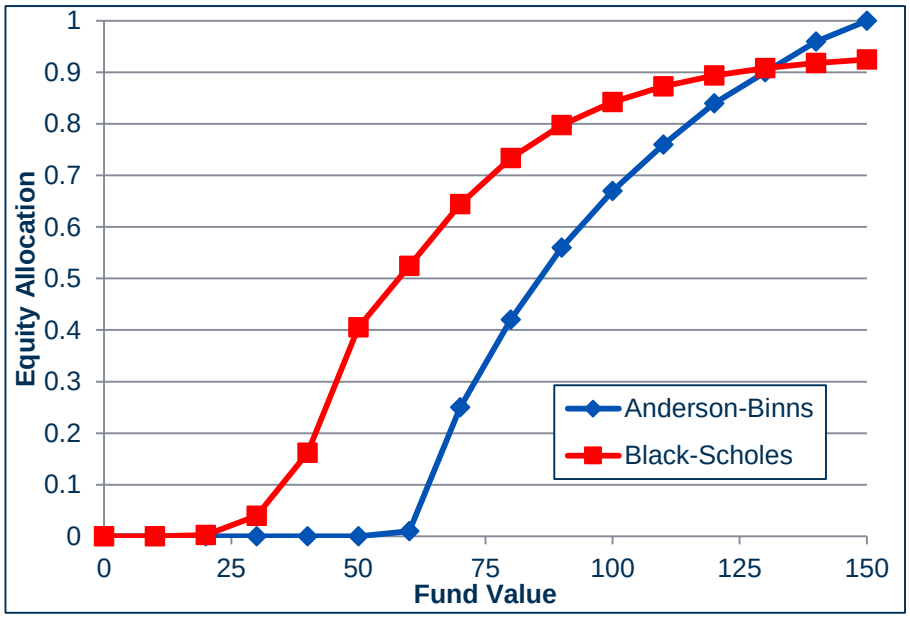
> "Suppose *k* is the maximum depreciation on present market values whish is envisaged. Estimate the value of a function we shall call the remainder, *R*, i.e. the excess of the total assets at market value over the liabilities on a gross premium basis…It can then be argued that *R* / *k* can safely be invested in equities provided the balance of the fund is reasonably matched.'

- Recknell approach was equivalent to $k = 1$
- Anderson and Binns advocated $k = 0.6$
- Sessional meeting discussion noted Dow Jones Industrial Index fell by over 80% between 1929 and 1932

Anderson and Binns and Dynamic Hedging / Portfolio Insurance

- The Anderson and Binns formula is the same as that used in modern 'constant proportion portfolio insurance' (CPPI) strategies
  - CPPI strategies are typically re-balanced daily

- A&B did not advocate continuous or daily re-balancing, but their explicit use of market values made it clear that the equity allocation implied by their formula would change over time



Black-Scholes generally permits higher EBR as it factors in ability to re-balance continuously in the future, whereas the A-B formula assumes no future re-balancing

Actuaries and market risk in the 1950s

- Both immunization and the Anderson-Binns equity allocation formula were early forms of risk management strategy based on managing the financial risks of a life office *dynamically* and *on a market value basis*

- These ideas pre-dated the rigorous development of dynamic hedging by Black, Scholes, Merton by a couple of decades and probably represented the most sophisticated financial market risk management thinking to be found anywhere at the time

- But there was little further development of actuarial financial risk management ideas of this sort in the 1960s-1990s
  - Indeed, explicit rejection of use of dynamic hedging ideas in the'70s and '80s

Risk-based capital and long-term financial guarantees in the late 20th century

Risk theory and British actuaries prior to 1980

- 'Risk theory' was first developed and applied to general insurance reserving
  - Stochastic modelling of the incidence of general insurance claims in order to assess evolution of reserves and probabilities of ruin
  - Mainly developed by Scandinavians, most notably in 1950s

- Generally found to be distasteful by British life actuaries, e.g. Ryder (1976):

> "Risk theory is a rather esoteric branch of actuarial theory which has been extensively developed by the more theoretical continental actuarial tradition.
>
> The practical actuary, however, finds that he hardly ever uses this theory."

- British life business continued to run on net premium valuations with asset yield discount rates and assets valued at lower of market value and book value
  - Non-diversifiable nature of financial market risk was not explicitly addressed anywhere in British actuarial reserving framework of 1950s - 1970s

Anticipations of probabilistic approaches to capital assessment for life business

- However, Jim Pegler (Institute President 1968-70), at Sessional meeting in discussion of Skelman (1968) raised the prospect of a probabilistic approach being used to avoid over-distribution of surplus as with-profit bonuses:

> "Theoretically, they could approach the problem from the point of view of the Theory of Risk and estimate the probability of ruin.
>
> But even supposing that a reasonably accurate numerical figure could be obtained, he doubted whether it was helpful in practice to consider whether a probability of, say, 1 in 1000 or 1 in 10,000 should be aimed at;
>
> He was choosing figures more or less at random, as he had little idea of the order of magnitude which was relevant."

Benjamin (1971 / 1976): Approach

- Sidney Benjamin was the first to develop quantitative results for a risk theory approach to long-term financial guarantees in life business

- His approach:
  - Capital requirement defined as a tail percentile of the discounted present value of maturity guarantee shortfall (net of invested premiums charged for guarantee)
  - Probability distribution generated by independently sampling from the annual equity returns generated over the 51 years from 1919 to 1970
  - Assumed a 2% probability of ruin, generated 50 simulation paths and used the one that produced the greatest reserve

- Methodologically, very quick and dirty

Benjamin (1971 / 1976): Results and Reaction

- Results implied that a ten-year money-back guarantee on a unit-linked equity fund required a starting capital requirement of c. 30% of the guarantee

> "Reserves…are unexpectedly high.
>
> The contract is probably not a commercial proposition."

- Results presented at Institute sessional meeting in 1971

- Meeting had to be converted by President into a 'private discussion' so as to avoid the debate being minuted
  - "By far the stormiest (sessional meeting) I have ever attended."
    - P. Smith in Corby (1977)

- Results eventually published five years later after presentation at International Congress of Actuaries in Tokyo

Scott (1977), Wilkie (1977)

- Further research on probabilistic estimation of capital requirements for unit-linked maturity guarantees then quickly followed
- Scott (1977) concluded there was evidence of mean-reversion in historical equity returns
  - Implemented by reducing annual volatility of independent returns from 19% to 10%
- Produced substantially lower estimates than Benjamin for capital requirement
- Wilkie (1977) developed embryonic Wilkie model, using auto-regressive stochastic processes
  - Like Scott, assumed significant mean-reversion, but effect was lower than that implied by Scott's adjustment
  - Also included data for 1973-76 (which was a highly volatile period)
- Wilkie's results closer to Benjamin's than Scott's

Corby (1977)

- Benjamin, Scott and Wilkie all assumed some form of stochastic process for equities and used it to derive a capital requirement based on probability of ruin
- Corby (1977) rejected this approach:

> "It is unlikely that [he] would be able to derive a model of stock market behaviour which would be satisfactory for extrapolation into the future and which would be generally acceptable as a basis for reserving."

- His alternative:

> "Assume a trend line for the performance of the index together with a spread about that line…assume that all purchases are made at the top of the range and all sales (i.e. maturities) at the bottom of the range."

$$Guarantee\ Shortfall\ at\ Maturity = G - \frac{1-k}{1+k}\sum(1+r)^t$$

- But *k* can only come from a model of the probability distribution of returns!
  - His calibration approach found *k* parameter value (of 0.4) that produced results similar to Benjamin's.

Corby (1977) – On Option Hedging Approach

- Paper featured the Institute's first explicit formal engagement with the Black-Scholes-Merton option pricing theory published in 1973

> "Here practice and theory are irreconcilable...the investment procedure [dynamic hedging] is a perfectly reasonable theoretical concept but as a practical proposition it is one which *contains risks greater than the risk which it is designed to eliminate*."

- There are undoubtedly many practical challenges and risks inherent in implementing a dynamic hedging strategy, but the italics was an unsubstantiated assertion presented without evidence or expansion

Maturity Guarantee Working Party (1980)

- Wilkie and Benjamin were Working Party members, Corby not, so methods and conclusions unsurprisingly consistent with their work, i.e.
  - Probability of ruin based on funding guarantee shortfalls as they fell due
  - Stochastic simulation model of equity returns used
  - Mean-reversion in equity returns embedded in equity model

- Results implied writing non-hedged long-term equity guarantees required uneconomical amounts of capital
- And Working Party explicitly ruled out taking credit for hedging in the capital assessment:

> "There is no basis for reducing maturity guarantee reserves because a company follows some form of immunization strategy…"

U-L Guarantees v. With-Profit Guarantees

- The 1980 Maturity Guarantee Working Party brought risk-based capital requirements into core actuarial reserving methods for unit-linked guarantees
- The preclusion of taking credit for hedging in capital requirements meant these products were uneconomical and insurers stopped providing these guarantees
- But with-profit guarantees were viewed as completely distinct from unit-linked guarantees
  - Actuarial discretion in bonus and investment strategy could mitigate guarantee risks
  - Accurate modelling of these actions viewed as very difficult and unnecessary
  - 'Countryside to explore on foot and not by fast car' – Redington (1976)
- Stochastic methods for setting with-profit capital requirements were not implemented until 21st century (following the GAO crisis)

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